

Security-Constrained Unit Commitment With AC/DC Transmission Systems

Azim Lotfjou, *Member, IEEE*, Mohammad Shahidehpour, *Fellow, IEEE*, Yong Fu, *Member, IEEE*, and Zuyi Li, *Member, IEEE*

Abstract—This paper presents the solution to the security-constrained unit commitment (SCUC) problem with a detailed representation of high voltage direct current (DC) transmission system with current source converters (CSCs). The SCUC problem is decomposed into a master problem for solving unit commitment (UC) problem and hourly transmission security check subproblems that evaluate branch flows and bus voltages of integrated AC/DC transmission systems. The solution of the transmission security check subproblem is based on a linear programming (LP) formulation that minimizes AC bus mismatches subject to AC/DC transmission constraints. The final SCUC solution prescribes an economic and secure operation and control strategy for AC/DC transmission systems and coordinates DC power transfers for enhancing the economics and the security of AC transmission systems. Numerical tests illustrate the efficiency of the proposed AC/DC transmission model.

Index Terms—AC/DC transmission systems, Benders decomposition, current source converters, security-constrained unit commitment.

NOMENCLATURE

b	Index for AC buses.
c	Index for contingencies.
h	Index for DC converters.
i	Index for generating units.
l	Index for transmission lines.
m	Index for AC bus terminals connected to converters.
$I_{dc,h}$	DC current of converter h .
$MP_{b,1}, MP_{b,2}$	Slack variables for the real power mismatch at bus b (≥ 0).
$MQ_{b,1}, MQ_{b,2}$	Slack variables for the reactive power mismatch at bus b (≥ 0).
NB	Number of AC buses.
P_m, Q_m	Real and reactive power withdrawals at bus m .
P_m^{inj}, Q_m^{inj}	Real and reactive power injections at bus m .

$P_{term,h}, Q_{term,h}$	Real and reactive power of converter h .
R_{dc}	Resistance of DC line.
T_h	Off-nominal tap ratio of the transformer connected to converter h .
$V_{dc,h}$	DC voltage of converter h .
V_m	AC bus voltage at bus m .
w	Objective value of SCUC subproblem.
$X_{c,h}$	Commutation reactance of converter h .
α_h	Firing/extinction angle of converter h .
ϕ_h	Power factor angle of converter h ($0 \leq \phi_h \leq (\pi)/(2)$ for rectifier, $-(\pi)/(2) \leq \phi_h \leq 0$ for inverter).
ΔP_m	Real power mismatch at bus m .
ΔQ_m	Reactive power mismatch at bus m .
$\Delta R1_h, \Delta R2_h, \Delta R3_h$	DC mismatches of converter h .
Δ_i	Permissible real power adjustment for unit i .
A	Bus-unit incidence matrix.
G_h	Admittance matrix of DC transmission system for converters h .
I	Unit state vector.
I_{dc}, ΔI_{dc}	DC current vector and its incremental vector.
J1, J2, J3, J4	Jacobian matrices.
MP1, MP2	Mismatch vector for real power slack variables.
MQ1, MQ2	Mismatch vector for reactive power slack variables.
P, ΔP	Real power generation vector and its increment.
Q_{max}, Q_{min}	Upper and lower limit vectors for reactive power generation.
V, V_{dc}	AC and DC bus voltage vectors.
x	Vector of SCUC variables.

Manuscript received June 05, 2009; revised July 10, 2009. First published December 31, 2009; current version published January 20, 2010. Paper no. TPWRS-00084-2009.

The authors are with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616 USA.

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TPWRS.2009.2036486

$\phi, \Delta\phi$	Power factor angle vector and its increment.
$\alpha, \Delta\alpha$	Firing/extinction angle vector and its increment.
$\delta, \Delta\delta$	Bus phase angle vector and its increment.
$\pi, \underline{\psi}, \bar{\psi}$	Simplex multiplier vectors.
dP_0	Initial AC bus real power mismatch vector.
dQ_0	Initial AC bus reactive power mismatch vector.
$dR1_0, dR2_0, dR3_0$	Initial mismatch vectors for DC equations.
$\Delta I_{dc,max}, \Delta I_{dc,min}$	Upper and lower limit vectors for incremental converter currents.
$\Delta PL_{ac}, \Delta PL_{dc}$	Increment vectors for AC and DC line flows.
$\Delta PL_{ac,max}, \Delta PL_{ac,min}$	Upper and lower limit vectors for incremental AC line flows.
$\Delta PL_{dc,max}, \Delta PL_{dc,min}$	Upper and lower limit vectors for incremental DC line flows.
ΔQ	Incremental vector of reactive power generation.
$\Delta Q_{max}, \Delta Q_{min}$	Upper and lower limit vectors for incremental unit reactive power.
ΔT	Increment vector for transformer taps.
$\Delta T_{max}, \Delta T_{min}$	Upper and lower limit vectors for incremental transformer taps.
$\Delta V, \Delta V_{dc}$	Increment vectors for AC and DC bus voltages.
$\Delta V_{max}, \Delta V_{min}$	Upper and lower limit vectors for incremental AC voltages.
$\Delta\gamma$	Increment vector for phase-shifter angles.
$\Delta\gamma_{max}, \Delta\gamma_{min}$	Upper and lower limit vectors for incremental phase-shifter angles.
$\Delta\alpha_{max}, \Delta\alpha_{min}$	Upper and lower limit vectors for incremental firing/extinction angles.
$\Delta\phi_{max}, \Delta\phi_{min}$	Upper and lower limit vectors for incremental power factor angles.

I. INTRODUCTION

DC transmission systems are often considered for delivering bulk power via long distances. The attributes of DC transmission systems include low capacitance, low average transmission cost in long distances, ability to prevent cascaded outages in AC transmission systems, rapid adjustments for direct power flow controls, ability to improve the stability of AC transmission systems, mitigation of transmission congestion, enhancement of transmission capacity, rapid frequency

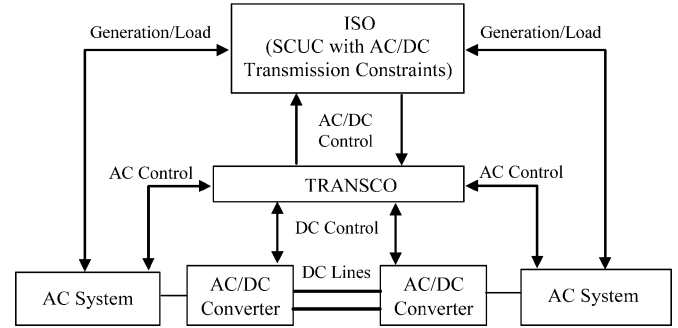


Fig. 1. Market clearing process with AC/DC transmission.

control following a loss of generation, ability to damp out regional power oscillations following major contingencies, and offering major economic incentives for supplying loads [1]–[4]. Flexible and fast DC controls provide efficient and desirable performance for a wide range of AC transmission systems [5].

Proper control modes applied to rectifiers (source) and inverters (sink) could maintain DC voltages near rated values and set power factors as high as possible. DC transmission system would require two control modes out of five for source and sink operations. The control modes include fixed firing/extinction advance angles, tap setting of converter transformers, DC currents, DC voltages, and DC power [6], [7]. The selection of DC control modes depends on the nature of specific applications for enhancing the economic and security.

DC transmission system configurations include monopolar, bipolar, tripolar, and multi-terminal which are used in diverse applications including the reinforcement of AC transmission flows and the interconnection of several AC transmission systems with different operating frequencies [8].

In the past, DC transmission systems were modeled in the optimal power flow (OPF) problem to analyze the impact of DC transmission systems on the operation of AC transmission systems [9], [10]. Several numerical techniques such as unified and sequential methods were considered for solving integrated AC/DC transmission flow equations [11]–[13]. The unified method solves AC and DC power flow equations simultaneously. The sequential method applies an iterative procedure between AC and DC transmission systems.

With the advent of electricity restructuring, DC transmission systems were utilized to maintain the system security and decrease social costs. The simplest approach to consider DC transmission systems in SCUC was to regard DC terminals as constant power injections or withdrawals in AC transmission systems [14]–[16]. However, such simplified models in SCUC may ignore the dependency of AC power flows to DC transmission variables.

Fig. 1 depicts the market clearing process for AC/DC transmission systems. The AC system includes AC generators, loads, and transmission lines. The DC transmission system is composed of converter terminals and DC transmission lines. At first, the independent system operator (ISO) receives supply/demand bids and collects AC/DC transmission data provided by transmission companies (TRANSCOs). Once the market is cleared, the ISO will send hourly schedules (unit commitment,

generation dispatch, and load schedules) to the AC system and optimal control strategies to the TRANSCOs which will activate control signals (tap-changing and phase-shifting transformers, adjustable shunt capacitors, firing angles of thyristors, and taps of converter transformers) to operate AC/DC transmission systems.

Since most DC transmission installations in the world use a line-commutation based current source converter (CSC) [17], we study in this paper the application of CSC-DC transmission system. The other option for the modeling of DC transmission system is based on the voltage source converter (VSC-DC) [18]–[20]. The application of VSC-DC to SCUC is under investigation and will appear in our subsequent publications. The proposed SCUC model would secure the power flow solution and optimize the hourly UC by a set of proper control strategies of AC/DC transmission systems. The SCUC solution for handling AC/DC transmission constraints applies Benders decomposition for mitigating transmission congestion and improving power system economics while satisfying system security constraints including transmission flow and bus voltage limits. Once a credible contingency occurs on AC/DC transmission lines or generating units, contingency subproblems will minimize transmission violations. In such cases, preventive actions will accommodate uncontrollable contingencies by transferring the base case operating point to a secure state. A corrective action will otherwise be considered for accommodating controllable contingencies [21].

The rest of the paper is organized as follows. Section II models the CSC (DC terminal) and power flow equations for the DC transmission system. The SCUC formulation with AC/DC transmission constraints and Benders decomposition based solution are presented in Section III. The proposed algorithm is tested with the IEEE 14-bus and IEEE 118-bus systems in Section IV. We summarize the conclusion in Section V.

II. DC TRANSMISSION SYSTEM MODEL

DC transmission systems consist of at least two converters (i.e., rectifiers and inverters) and overhead lines that link converters. In order to eliminate harmonics generated by AC/DC systems, AC filters are installed on the AC side of DC terminals. In this section, we review DC transmission systems and corresponding DC power flow equations for our SCUC formulation.

A. CSC Model

Fig. 2 presents a typical CSC system which is connected to AC bus m through a coupling transformer. In order to model the CSC h which is linked to the AC bus m , five converter variables, i.e., $V_{dc,h}$, $I_{dc,h}$, T_h , α_h , and ϕ_h are considered in Fig. 2. These variables determine the DC line operating state.

A converter, either a rectifier or an inverter, is modeled by (1)–(3). The converter equations (1) and (2) express $V_{dc,h}$ in terms of V_m and other converter variables while the coupling transformer is assumed to be lossless. The converter equation (3), which can be expressed using a nodal-mesh procedure, represents the DC voltage-current relationship which depends

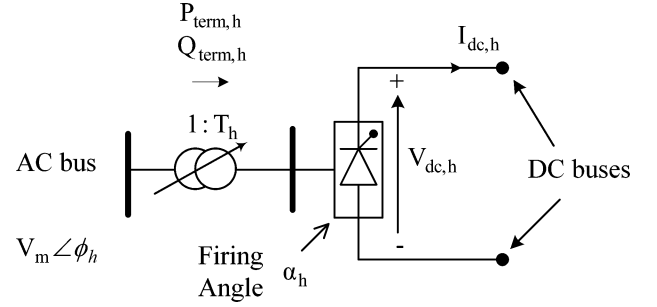


Fig. 2. Schematic diagram of a CSC-DC system.

on the DC transmission system configuration (i.e., monopole, bipole, tri-pole, or a multi-terminal link):

$$V_{dc,h} = \frac{3\sqrt{2}}{\pi} T_h V_m \cos \alpha_h - \frac{3}{\pi} X_{c,h} I_{dc,h} \text{ rectifier} \quad (1)$$

$$V_{dc,h} = -\frac{3\sqrt{2}}{\pi} T_h V_m \cos \alpha_h + \frac{3}{\pi} X_{c,h} I_{dc,h} \text{ inverter}$$

$$V_{dc,h} = 0.995 \frac{3\sqrt{2}}{\pi} T_h V_m \cos \phi_h \text{ rectifier} \quad (2)$$

$$V_{dc,h} = -0.995 \frac{3\sqrt{2}}{\pi} T_h V_m \cos \phi_h \text{ inverter}$$

$$f(\mathbf{V}_{dc}, I_{dc,h}) = 0. \quad (3)$$

Here, (3) for converter h is rewritten as (4) in a matrix form:

$$I_{dc,h} - \mathbf{G}_h \cdot \mathbf{V}_{dc} = \mathbf{0} \quad (4)$$

where \mathbf{G}_h is the admittance matrix of DC transmission system for converter h . A DC terminal is selected as slack bus in each DC transmission system. Similar to that in an AC transmission system, the slack bus balances the real power among DC terminals in the DC transmission system [22].

B. Control Modes of DC Transmission Systems

Each CSC (rectifier or inverter) is regulated by two out of five control modes. The modes are as follows.

- 1) Constant current (CC)

$$I_{dc,h} = \text{const}$$

- 2) Constant voltage (CV)

$$V_{dc,h} = \text{const}$$

- 3) Constant power (CP)

$$P_{\text{term},h} = V_{dc,h} I_{dc,h} = \text{const}$$

- 4) Constant firing/extinction angle (CFA/CEA)

$$\alpha_h = \text{const}$$

5) Constant transformer tap ratio (CTR)

$$T_h = \text{const}$$

The selection of two control modes would maximize the economic benefits of DC transmission systems while keeping all variables within their limits. The rules for selecting effective CSC control modes are given below [1]–[3],

- Rule 1) At least one converter in each DC transmission system would select CV as the first control mode; other converters would maintain CC as the first control mode. If converters are not operating at their minimum firing/extinction angles, CP may be applied as the second control mode.
- Rule 2) If a rectifier is not operating at its minimum firing angle, the rectifier would be operated as (CC/CP plus CV). For an inverter which is not operating at its minimum extinction angle, (CC/CP/CV plus CEA) is selected.
- Rule 3) Once the minimum firing angle of rectifier is reached, the rectifier would switch from (CC plus CV) to (CFA plus CTR). In this case, the inverter would maintain CC plus CEA.
- Rule 4) If an inverter is operating at its minimum extinction advance angle, it would switch to (CC/CP/CV plus CTR).

C. Power Flow Equations of DC Transmission Systems

Using the per-unit system given in Appendix A, nodal power balance equations at the AC bus m that is linked to converter h are listed as

$$\Delta P_m = P_m^{\text{inj}} - P_m - P_{\text{term},h} = 0 \quad (5)$$

$$\Delta Q_m = Q_m^{\text{inj}} - Q_m - Q_{\text{term},h} = 0 \quad (6)$$

where

$$P_{\text{term},h} = V_{\text{dc},h} I_{\text{dc},h} \quad (7)$$

$$Q_{\text{term},h} = V_{\text{dc},h} I_{\text{dc},h} \tan \phi_h. \quad (8)$$

CSC always consumes reactive power and $Q_{\text{term},h}$ is non-negative. Similarly, (1), (2) and (4) are replaced in pu as

$$\begin{cases} \Delta R1_h = V_{\text{dc},h} - T_h V_m \cos \alpha_h + X_{c,h} I_{\text{dc},h} = 0 & \text{rectifier} \\ \Delta R1_h = V_{\text{dc},h} + T_h V_m \cos \alpha_h - X_{c,h} I_{\text{dc},h} = 0 & \text{inverter} \end{cases} \quad (9)$$

$$\begin{cases} \Delta R2_h = V_{\text{dc},h} - 0.995 T_h V_m \cos \phi_h = 0 & \text{rectifier} \\ \Delta R2_h = V_{\text{dc},h} + 0.995 T_h V_m \cos \phi_h = 0 & \text{inverter} \end{cases} \quad (10)$$

$$\Delta R3_h = I_{\text{dc},h} - \mathbf{G}_h \mathbf{V}_{\text{dc}} = 0. \quad (11)$$

In the next section, we will apply the Newton–Raphson method that utilizes the linearized version of (5)–(11) for the network security violation checking in base case and contingency conditions.

III. SCUC WITH DC CONSTRAINTS

A. Formulation of SCUC With AC/DC Constraints

SCUC is generalized in (12) where $f(\mathbf{x})$ is composed of production cost, and start up and shut down costs of individual units for the study horizon [21], [23]–[25]:

$$\begin{aligned} & \text{Min} f(\mathbf{x}) \\ & \text{S.t.} \\ & \mathbf{g1}(\mathbf{x}) \leq \mathbf{b1} \\ & \mathbf{ge1}(\mathbf{x}) = \mathbf{be1} \\ & \mathbf{g2}(\mathbf{x}) \leq \mathbf{b2} \\ & \mathbf{ge2}(\mathbf{x}) = \mathbf{be2} \end{aligned} \quad (12)$$

where \mathbf{x} is composed of on/off status and generation of generating units, startup and shutdown indicators, and AC/DC transmission system variables. The first set of inequality constraints $\mathbf{g1}(\mathbf{x}) \leq \mathbf{b1}$ and equality constraints $\mathbf{ge1}(\mathbf{x}) = \mathbf{be1}$ represents UC constraints such as

- 1) Power balance;
- 2) Generating unit capacity;
- 3) System spinning and operating reserve requirements;
- 4) Ramping up/down limits;
- 5) Minimum up/down time limits;
- 6) Maximum number of simultaneous on/off in a plant;
- 7) Maximum number of on/off of a unit in a given period;
- 8) Fuel and multiple emission limits;
- 9) AC/DC power flow equations;
- 10) Limits on AC/DC control variables including real and reactive power generations, controlled shunt capacitors, tap-changing and phase-shifting transformers, firing/extinction angles and coupling transformer tap ratios of converters;
- 11) AC/DC network security constraints including AC/DC transmission flow and bus voltage limits, and limits to DC currents, voltages and power of converters;
- 12) Time limited corrective controls, such as permissible real power adjustments, for handling contingencies.

B. Solution of SCUC With AC/DC Constraints

We apply the Benders decomposition to solve the SCUC with AC/DC transmission systems as depicted in Fig. 3. The master problem applies MIP for solving the UC problem. The Newton–Raphson based subproblem is utilized to check transmission flows and bus voltages in base case and contingency conditions. The SCUC subproblem considers both real and reactive power mismatches at AC buses which are minimized based on optimal adjustments of scheduled power generation, tap-changing transformers, and phase shifters in the AC system, and by adjusting DC transmission controls (e.g., converter firing/extinction advance angles and transformer tap settings). The subproblem generates Benders cuts for the next UC iteration once the mismatch exists in either base case or

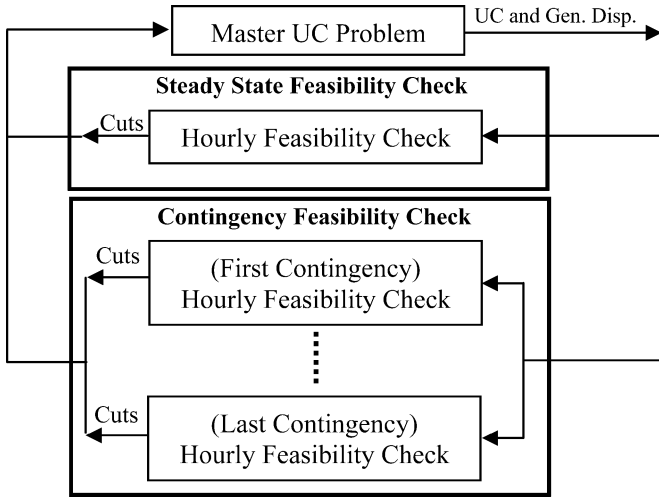


Fig. 3. SCUC with AC/DC transmission systems.

contingencies. This iterative procedure will continue until all mismatches are removed.

C. SCUC Subproblem for Base Case

The subproblem for base case is formulated as (13)–(26). (14)–(26) represents AC/DC power flow equations, limits on real and reactive power generations, AC and DC line flows, real power withdrawals at AC converter buses, AC bus voltages magnitudes, transformer tap settings, phase shifter angles, DC bus voltages, DC voltages and currents of converters, converter firing/extinction advance angles, and converter power factor (lagging/leading) angles, respectively. Note that (15) is for all generating units except the ones at the slack bus. Appendix B lists the elements of Jacobian matrices $\mathbf{J1}$, $\mathbf{J2}$, $\mathbf{J3}$, and $\mathbf{J4}$ for DC transmission systems:

$$\text{Min } w = \sum_{b=1}^{NB} (MP1_b + MP2_b + MQ1_b + MQ2_b) \quad (13)$$

$$\begin{bmatrix} \mathbf{A} \cdot \Delta \mathbf{P} \\ \mathbf{A} \cdot \Delta \mathbf{Q} \\ 0 \\ 0 \\ 0 \end{bmatrix} - [\mathbf{J1}] \begin{bmatrix} \Delta \delta \\ \Delta \mathbf{V} \\ \Delta \mathbf{T} \\ \Delta \gamma \\ \Delta \mathbf{V}_{dc} \\ \Delta \mathbf{I}_{dc} \\ \Delta \alpha \\ \Delta \phi \end{bmatrix} + \begin{bmatrix} MP1 \\ MQ1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} MP2 \\ MQ2 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} dP_0 \\ dQ_0 \\ dR1_0 \\ dR2_0 \\ dR3_0 \end{bmatrix} \quad (14)$$

$$\Delta \mathbf{P} = 0 \quad \pi \quad (15)$$

$$\Delta \mathbf{Q}_{\min} \leq \Delta \mathbf{Q} \leq \Delta \mathbf{Q}_{\max} \quad \underline{\psi}, \bar{\psi} \quad (16)$$

$$\Delta \mathbf{P}_{Lac, \min} \leq \Delta \mathbf{P}_{Lac} = [\mathbf{J2}] \begin{bmatrix} \Delta \delta \\ \Delta \mathbf{V} \\ \Delta \mathbf{T} \\ \Delta \gamma \end{bmatrix} \leq \Delta \mathbf{P}_{Lac, \max} \quad (17)$$

$$\Delta \mathbf{P}_{Ldc, \min} \leq \Delta \mathbf{P}_{Ldc} = [\mathbf{J3}] \begin{bmatrix} \Delta \mathbf{V}_{dc} \\ \Delta \mathbf{I}_{dc} \end{bmatrix} \leq \Delta \mathbf{P}_{Ldc, \max} \quad (18)$$

$$\Delta \mathbf{P}_{term, \min} \leq \Delta \mathbf{P}_{term} = [\mathbf{J4}] \begin{bmatrix} \Delta \mathbf{V}_{dc} \\ \Delta \mathbf{I}_{dc} \end{bmatrix} \leq \Delta \mathbf{P}_{term, \max} \quad (19)$$

$$\Delta \mathbf{V}_{\min} \leq \Delta \mathbf{V} \leq \Delta \mathbf{V}_{\max} \quad (20)$$

$$\Delta \mathbf{T}_{\min} \leq \Delta \mathbf{T} \leq \Delta \mathbf{T}_{\max} \quad (21)$$

$$\Delta \gamma_{\min} \leq \Delta \gamma \leq \Delta \gamma_{\max} \quad (22)$$

$$\Delta \mathbf{V}_{dc, \min} \leq \Delta \mathbf{V}_{dc} \leq \Delta \mathbf{V}_{dc, \max} \quad (23)$$

$$\Delta \mathbf{I}_{dc, \min} \leq \Delta \mathbf{I}_{dc} \leq \Delta \mathbf{I}_{dc, \max} \quad (24)$$

$$\Delta \alpha_{\min} \leq \Delta \alpha \leq \Delta \alpha_{\max} \quad (25)$$

$$\Delta \phi_{\min} \leq \Delta \phi \leq \Delta \phi_{\max} \quad (26)$$

The steps for solving the SCUC subproblem include

- Step 1) Calculate elements of corresponding Jacobian matrices $\mathbf{J1}$, $\mathbf{J2}$, $\mathbf{J3}$, and $\mathbf{J4}$, initial AC bus mismatch vectors dP_0 and dQ_0 , and initial DC power mismatch $dR1_0$, $dR2_0$, and $dR3_0$. The calculation is based on the initial UC solution and the given AC/DC transmission system state.
- Step 2) Use LP to minimize (13) and calculate changes in the AC/DC transmission state and control variables.
- Step 3) Update state and control variables. Recalculate elements of Jacobian matrices and mismatch vectors.
- Step 4) Minimize (13) and calculate changes in AC/DC transmission system state and control variables. If the difference between the last two iterative changes is less than a specified tolerance, stop the process. Otherwise, go back to Step 3.

If w is larger than zero, a mismatch Benders cut (27) will be formed and added to the UC problem for calculating the next iterative solution of master problem:

$$w + \pi(\mathbf{P} - \hat{\mathbf{P}}) + \bar{\psi} \mathbf{Q}_{\max}(\mathbf{I} - \hat{\mathbf{I}}) - \underline{\psi} \mathbf{Q}_{\min}(\mathbf{I} - \hat{\mathbf{I}}) \leq 0 \quad (27)$$

where $\hat{\mathbf{I}}$ and $\hat{\mathbf{P}}$ are the UC and generation dispatch results in the current iteration, respectively.

D. SCUC Subproblem for Contingencies

The subproblem for a contingency c is formulated in (28)–(41). The objective function (28) is introduced according to the Newton–Raphson method for minimizing real and reactive bus power mismatches and calculating a converged AC power flow solution subject to transmission flow and bus voltage limits.

In each contingency case, corrective and preventive actions are considered for managing AC/DC violations. Corrective actions refer to the redispatch of generating units and adjustments of transmission flow controls for mitigating transmission flow violations in real time. Preventive actions refer to day-ahead adjustments of transmission flows by applying transformer controls, as well as UC and economic dispatch (ED) of generating units. Preventive actions will be applied if corrective actions are

not feasible in real time. (30) represents the physically acceptable corrective action for generating units [25]:

$$\begin{aligned} \text{Min } w^c = & \sum_{b=1}^{\text{NB}} (MP1_b^c + MP2_b^c \\ & + MQ1_b^c + MQ2_b^c) \end{aligned} \quad (28)$$

$$\begin{bmatrix} \mathbf{A} \cdot \Delta \mathbf{P}^c \\ \mathbf{A} \cdot \Delta \mathbf{Q}^c \\ 0 \\ 0 \\ 0 \end{bmatrix} - [\mathbf{J1}] \begin{bmatrix} \Delta \delta^c \\ \Delta V^c \\ \Delta T^c \\ \Delta \gamma^c \\ \Delta V_{dc}^c \\ \Delta I_{dc}^c \\ \Delta \alpha^c \\ \Delta \phi^c \end{bmatrix} + \begin{bmatrix} MP1^c \\ MQ1^c \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} MP2^c \\ MQ2^c \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} dP_0^c \\ dQ_0^c \\ dR1_0^c \\ dR2_0^c \\ dR3_0^c \end{bmatrix} \quad (29)$$

$$|\mathbf{P}^c - \hat{\mathbf{P}}^0| \leq \Delta \bar{\pi}^c, \underline{\pi}^c \quad (30)$$

$$\Delta Q_{\min}^c \leq \Delta Q^c \leq \Delta Q_{\max}^c \quad \underline{\psi}^c, \bar{\psi}^c \quad (31)$$

$$\Delta PL_{ac,\min}^c \leq \Delta PL_{ac}^c \leq \Delta PL_{ac,\max}^c \quad (32)$$

$$\Delta PL_{dc,\min}^c \leq \Delta PL_{dc}^c \leq \Delta PL_{dc,\max}^c \quad (33)$$

$$\Delta P_{\text{term},\min}^c \leq \Delta P_{\text{term}}^c \leq \Delta P_{\text{term},\max}^c \quad (34)$$

$$\Delta V_{\min}^c \leq \Delta V^c \leq \Delta V_{\max}^c \quad (35)$$

$$\Delta T_{\min}^c \leq \Delta T^c \leq \Delta T_{\max}^c \quad (36)$$

$$\Delta \gamma_{\min}^c \leq \Delta \gamma^c \leq \Delta \gamma_{\max}^c \quad (37)$$

$$\Delta V_{dc,\min}^c \leq \Delta V_{dc}^c \leq \Delta V_{dc,\max}^c \quad (38)$$

$$\Delta I_{dc,\min}^c \leq \Delta I_{dc}^c \leq \Delta I_{dc,\max}^c \quad (39)$$

$$\Delta \alpha_{\min}^c \leq \Delta \alpha^c \leq \Delta \alpha_{\max}^c \quad (40)$$

$$\Delta \phi_{\min}^c \leq \Delta \phi^c \leq \Delta \phi_{\max}^c \quad (41)$$

If w_c is larger than zero, a mismatch Benders cut (42) for contingency c will be formed and added to the master UC problem for calculating the next iterative solution:

$$w^c + \pi^c (\mathbf{P} - \hat{\mathbf{P}}^0) + \bar{\psi}^c \mathbf{Q}_{\max} (\mathbf{I} - \hat{\mathbf{I}}^0) - \underline{\psi}^c \mathbf{Q}_{\min} (\mathbf{I} - \hat{\mathbf{I}}^0) \leq 0. \quad (42)$$

IV. NUMERICAL EXAMPLES

The modified IEEE 14-bus system and the IEEE 118-bus system are studied. Optimal control strategies for DC transmission systems are determined based on SCUC results. We set the mismatch tolerance at 0.001 pu (or 0.1 MVA). Numerical examples presented in this section are mainly to show the economic benefits of DC system while maintaining the system security in base case and contingency conditions.

A. IEEE 14-Bus System

The IEEE 14-bus system depicted in Fig. 4 is considered with pertinent data listed in Appendix C. This system has 5 generating units, 20 branches, 11 loads and 3 transformers. Bus 1 is

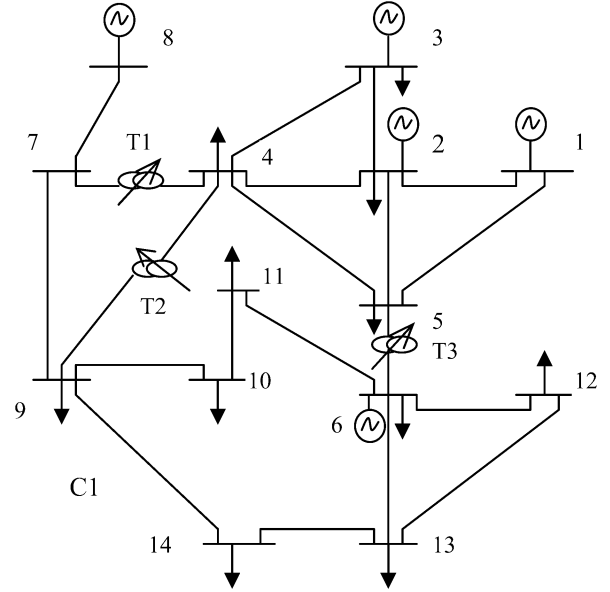


Fig. 4. IEEE 14-bus system.

selected as slack bus. The following cases are considered to examine the efficiency of the proposed model.

Case 0: UC solution without transmission constraints

Case 1: SCUC solution with AC transmission constraints

Case 2: ED solution when line 1–5 is replaced with a two-terminal DC line

Case 3: SCUC solution for Case 2

Case 4: SCUC for Case 1 considering the outage of line 2–3

Case 5: SCUC for Case 3 considering the outage of line 2–3

Case 6: SCUC for Case 3 with a simplified DC model (fixed power injection model).

The above cases are discussed as follows:

Case 0: When AC transmission constraints are not considered, the cheapest generating unit 1 is committed at all hours to supply the base load. The cheap units 2 and 3 with higher bidding prices of 10\$/MW and 18\$/MW are committed at certain hours to serve system loads. The expensive units 4 and 5 are always off. The corresponding hourly UC and ED results are listed in Tables I and II with an operating cost of \$45 666.28.

Case 1: In this case, the impact of AC transmission constraints on UC results is studied. The network security checking subproblem (13)–(26) is solved over 24 hours. The final SCUC solution is obtained after 6 iterations between master UC problem and network security checking subproblem. Tables III and IV show the hourly UC and ED in which UC changes in comparison to Case 0 are shown in bold. For example, the expensive generating unit 4 is committed at hours 1, 2, 7–24 to remove violations. We observed that the AC line 1–2 is the most congested line at peak-load hours because the cheapest unit 1 is dispatched at its maximum. In Table IV, unit 1 will pick up system losses. For example, at hour 1, the total generation dispatch is 184.16 MW which satisfies the system load of 181.3 MW

TABLE I
UC IN CASE 0

Unit	Hours (1-24)
1	111111111111111111111111111111
2	111100111111111111111111111111
3	000000011111110111111111111111
4	000000000000000000000000000000
5	000000000000000000000000000000

TABLE III
UC IN CASE 1

Unit	Hours (1-24)
1	111111111111111111111111111111
2	111111111111111111111111111111
3	11101 111111111111111111111111
4	110000 111111111111111111111111
5	000000000000000000001001111100

TABLE II
UNIT GENERATION DISPATCH IN CASE 0

Hour	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	150.00	31.30	0.00	0.00	0.00
2	150.00	20.94	0.00	0.00	0.00
3	130.22	20.00	0.00	0.00	0.00
4	103.60	0.00	0.00	0.00	0.00
5	129.50	0.00	0.00	0.00	0.00
6	135.40	20.00	0.00	0.00	0.00
7	150.00	31.30	0.00	0.00	0.00
8	150.00	40.02	12.00	0.00	0.00
9	150.00	50.00	12.38	0.00	0.00
10	150.00	50.00	27.92	0.00	0.00
11	150.00	50.00	30.51	0.00	0.00
12	150.00	50.00	17.56	0.00	0.00
13	150.00	45.20	12.00	0.00	0.00
14	150.00	46.84	0.00	0.00	0.00
15	150.00	50.00	27.92	0.00	0.00
16	150.00	50.00	33.10	0.00	0.00
17	150.00	50.00	20.15	0.00	0.00
18	150.00	50.00	30.51	0.00	0.00
19	150.00	50.00	43.46	0.00	0.00
20	150.00	50.00	53.82	0.00	0.00
21	150.00	50.00	59.00	0.00	0.00
22	150.00	50.00	33.10	0.00	0.00
23	150.00	50.00	25.33	0.00	0.00
24	150.00	50.00	12.38	0.00	0.00

TABLE IV
UNIT GENERATION DISPATCH IN CASE 1

Hour	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	83.21	50.00	40.95	10.00	0.00
2	82.54	50.00	31.09	10.00	0.00
3	82.65	50.00	20.26	0.00	0.00
4	77.14	28.44	0.00	0.00	0.00
5	80.11	39.68	12.00	0.00	0.00
6	82.98	50.00	25.15	0.00	0.00
7	80.11	50.00	40.95	10.00	0.00
8	84.50	50.00	60.60	10.00	0.00
9	85.20	50.00	70.38	10.00	0.00
10	82.76	50.00	68.29	30.21	0.00
11	82.43	50.00	68.37	33.08	0.00
12	84.08	50.00	73.21	13.59	0.00
13	84.87	50.00	65.41	10.00	0.00
14	84.17	50.00	55.64	10.00	0.00
15	82.76	50.00	68.29	30.21	0.00
16	85.20	50.00	80.00	11.03	10.00
17	83.93	50.00	69.02	20.43	0.00
18	82.43	50.00	68.37	33.08	0.00
19	83.91	50.00	80.00	22.77	10.00
20	82.64	50.00	80.00	34.48	10.00
21	82.00	50.00	80.00	40.36	10.00
22	85.20	50.00	80.00	11.03	10.00
23	83.25	50.00	69.11	26.30	0.00
24	85.20	50.00	70.38	10.00	0.00

and the loss of 2.86 MW. The operating cost in this case increases to \$64 086.07.

Case 2: In this case, line 1–5 is replaced with a two-terminal DC line shown in Fig. 5. The DC transmission data are listed in Tables XVIII–XX. We solve the ED problem based on UC results in Case 1 and find that a feasible ED cannot be obtained at certain hours. For instance, at hour 10, there is a reactive power mismatch of 2.14 MVar at bus 8. One possible reason is that the DC converters in this case consume additional reactive power and the committed units cannot provide enough reactive power. One option for calculating a feasible ED is to commit the most expensive unit 5 at bus 8 to supply additional reactive power. A new UC is calculated in Case 3.

Case 3: A new SCUC is calculated to examine the impact of the DC line flows. Tables V and VI show the UC and ED results. The bold numbers show the difference between Cases 3 and 1. Compared with Case 1, the more expensive unit 3 is off at hours 3 and 6 to reduce the operating cost since the DC transmission system allows the cheapest unit 1 to increase its generation. In fact, the DC line can transfer additional power (compared to the AC line 1–5 flow in Case 1) from bus 1 to bus 5 which results in the mitigation of congestion on line 1–2 and the additional dispatch of cheapest unit 1. Consequently, it is not necessary to turn on the more expensive unit 3 for supplying the load at hour 3. In addition, the expensive units 4 and 5 are committed to maintain bus voltages within their limits. In this case, the system operating cost drops to \$61 212.71 which is reduced

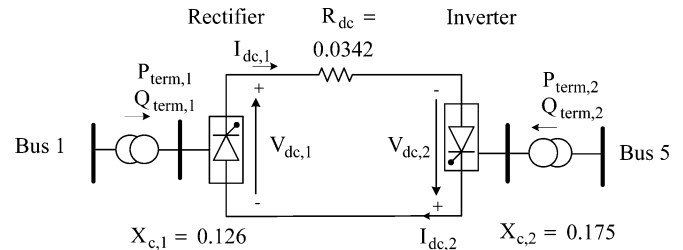


Fig. 5. Two-terminal DC transmission line.

TABLE V
UC IN CASE 3

Unit	Hours (1-24)
1	111111111111111111111111111111
2	111111111111111111111111111111
3	110010 1111111111111111111111
4	11100 111111111111111111111111
5	0000000 1111110 11111111111111

by \$64 086.07 – \$61 212.71 = \$2873.36 as compared with Case 1.

Table VII shows the hourly bus power mismatch per iteration. Bus power mismatches have decreased significantly at the second SCUC iteration in comparison with those at the first iteration. At the third SCUC iteration, the bus power mismatch at certain hours is smaller than the tolerance of 0.1 MVA. Consequently, no Benders cut are generated at those hours and the

TABLE VI
GENERATION DISPATCH IN CASE 3

Hour	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	94.69	50.00	17.56	22.43	0.00
2	100.20	50.00	14.18	10.00	0.00
3	90.35	50.00	0.00	13.09	0.00
4	85.62	20.00	0.00	0.00	0.00
5	93.61	26.32	12.00	0.00	0.00
6	96.67	50.00	0.00	12.32	0.00
7	94.69	50.00	17.56	22.43	0.00
8	104.99	50.00	30.45	10.00	10.00
9	106.28	50.00	39.61	10.00	10.00
10	108.22	50.00	53.28	10.00	10.00
11	108.49	50.00	55.63	10.00	10.00
12	106.93	50.00	44.16	10.00	10.00
13	105.64	50.00	35.04	10.00	10.00
14	96.07	50.00	43.98	10.00	0.00
15	108.22	50.00	53.28	10.00	10.00
16	108.86	50.00	57.86	10.00	10.00
17	107.20	50.00	46.50	10.00	10.00
18	108.49	50.00	55.63	10.00	10.00
19	110.09	50.00	67.08	10.00	10.00
20	109.34	50.00	75.06	13.19	10.00
21	106.80	50.00	77.72	18.20	10.00
22	108.86	50.00	57.86	10.00	10.00
23	107.85	50.00	51.06	10.00	10.00
24	106.28	50.00	39.61	10.00	10.00

TABLE VII
HOURLY NODAL MISMATCH PER ITERATION OF SCUC (MVA) IN CASE 3

Hour	Iter1	Iter2	Iter3	Iter4	Iter5	Iter6	Iter7
1	106.00	28.34	13.24	3.09	0.28	<0.1	<0.1
2	41.90	45.43	17.97	<0.1	4.21	<0.1	<0.1
3	67.29	11.05	4.38	13.37	<0.1	3.59	<0.1
4	0.35	<0.1	<0.1	<0.1	<0.1	<0.1	<0.1
5	22.75	5.32	3.31	6.63	2.77	0.28	<0.1
6	27.87	25.70	5.36	15.48	1.55	<0.1	<0.1
7	106.00	28.34	13.24	3.09	0.28	<0.1	<0.1
8	44.61	10.33	20.10	8.41	<0.1	<0.1	<0.1
9	47.99	29.36	10.25	0.17	<0.1	<0.1	<0.1
10	50.44	25.74	13.46	0.15	<0.1	<0.1	<0.1
11	50.91	24.92	14.19	0.24	<0.1	<0.1	<0.1
12	48.88	28.27	10.60	0.19	<0.1	<0.1	<0.1
13	46.30	33.25	15.13	15.22	0.47	0.42	<0.1
14	56.33	42.80	15.18	17.70	<0.1	<0.1	<0.1
15	50.44	25.74	13.46	0.15	<0.1	<0.1	<0.1
16	51.38	24.21	14.90	0.12	<0.1	<0.1	<0.1
17	49.34	27.62	11.31	0.31	<0.1	<0.1	<0.1
18	50.91	24.92	14.19	0.24	<0.1	<0.1	<0.1
19	53.33	21.02	18.42	0.18	<0.1	<0.1	<0.1
20	55.05	18.28	18.34	0.23	0.16	<0.1	<0.1
21	58.34	22.78	0.78	0.35	<0.1	<0.1	<0.1
22	51.38	24.21	14.90	0.12	<0.1	<0.1	<0.1
23	49.97	26.55	12.75	0.12	<0.1	<0.1	<0.1
24	47.99	29.36	10.25	0.17	<0.1	<0.1	<0.1

number of cuts have decreased from 24 at iteration 1 to 7 at iteration 5. There are no mismatches after 7 iterations. The economic control strategy of DC transmission system is devised based on the optimal operation of converters shown in Tables VIII and IX. This control scheme would be specified for each rectifier/inverter terminal in order to maintain two out of five control modes. Table X shows a set of 24-hour control strategies for the DC transmission system. At hour 1, the rectifier is operating at its minimum firing angle of $\alpha_r = 5$ Deg (in Table VIII) and, according to Rules 1 and 3, the first control mode is CFA = 5 Deg and the rectifier maintains $CTR = 1.09$ pu. At this hour, the inverter maintains $CC = 129$ A and $CV = -1.10$ pu.

The control mode at hours 2–4 is the same as that at hour 1. At hour 5, neither the rectifier firing angle nor the inverter extinction angle reaches its minimum. According to Rules 1 and 2,

TABLE VIII
RECTIFIER OPERATION STATUS IN CASE 3

Hour	P_{term} (MW)	Q_{term} (MVar)	V_{dc} (pu)	I_{dc} (A)	α (Deg)	T (pu)
1	44.69	13.42	1.11	129	5.00	1.09
2	50.20	16.00	1.12	145	5.00	1.10
3	40.35	11.93	1.07	121	5.00	1.05
4	35.62	9.56	1.11	104	5.00	1.08
5	43.59	13.60	1.08	130	6.08	1.06
6	46.67	14.32	1.11	135	5.00	1.09
7	44.69	13.42	1.11	129	5.00	1.09
8	54.99	18.51	1.11	160	5.00	1.10
9	56.28	19.22	1.11	164	5.00	1.10
10	58.22	20.32	1.10	170	5.00	1.10
11	58.48	20.55	1.10	171	5.39	1.10
12	56.94	19.58	1.10	166	5.00	1.10
13	55.63	18.93	1.10	162	5.00	1.10
14	46.07	14.05	1.11	133	5.00	1.09
15	58.22	20.32	1.10	170	5.00	1.10
16	58.87	20.68	1.10	172	5.00	1.10
17	57.20	19.77	1.11	166	5.28	1.10
18	58.48	20.55	1.10	171	5.39	1.10
19	60.09	21.35	1.10	176	5.00	1.10
20	59.33	20.86	1.10	173	5.00	1.10
21	56.80	19.45	1.11	165	5.00	1.10
22	58.87	20.68	1.10	172	5.00	1.10
23	57.84	20.15	1.10	169	5.30	1.10
24	56.28	19.22	1.11	164	5.00	1.10

TABLE IX
INVERTER OPERATION STATUS IN CASE 3

Hour	P_{term} (MW)	Q_{term} (MVar)	V_{dc} (pu)	I_{dc} (A)	α (Deg)	T (pu)
1	-44.14	10.16	-1.10	129	5.00	1.12
2	-49.51	12.10	-1.10	145	5.00	1.13
3	-39.86	9.01	-1.06	121	5.00	1.07
4	-35.26	7.68	-1.10	104	5.00	1.12
5	-43.03	11.03	-1.07	130	6.08	1.09
6	-46.07	10.84	-1.10	135	5.00	1.13
7	-44.14	10.16	-1.10	129	5.00	1.12
8	-54.14	14.00	-1.09	160	5.00	1.12
9	-55.40	14.53	-1.09	164	5.00	1.12
10	-57.26	15.36	-1.08	170	5.00	1.12
11	-57.52	15.45	-1.08	171	5.39	1.12
12	-56.03	14.80	-1.09	166	5.00	1.12
13	-54.76	14.31	-1.09	162	5.00	1.11
14	-45.49	10.63	-1.10	133	5.00	1.12
15	-57.26	15.36	-1.08	170	5.00	1.12
16	-57.89	15.63	-1.08	172	5.00	1.11
17	-56.29	14.89	-1.09	166	5.28	1.12
18	-57.52	15.45	-1.08	171	5.39	1.12
19	-59.07	16.13	-1.08	176	5.00	1.11
20	-58.35	15.76	-1.09	173	5.00	1.12
21	-55.90	14.70	-1.09	165	5.00	1.12
22	-57.89	15.63	-1.08	172	5.00	1.11
23	-56.91	15.16	-1.09	169	5.30	1.12
24	-55.40	14.53	-1.09	164	5.00	1.12

the rectifier would maintain $CC = 130$ A. The second rectifier control mode is $CV = 1.08$ pu. At this hour, the first inverter control mode is $CAE = 6.08$ Deg and its second control mode is $CP = -43.03$ MW. The alternative second inverter control modes are $CTR = 1.09$ pu or $CV = -1.07$ pu.

Case 4: Using the UC results in Table III, the system state can be transferred from base case to a new state in the event of line 2–3 outage (CTGL2-3) without any changes in UC. Only the generating unit dispatch is changed in accordance with (30). This is a controllable contingency and only corrective actions (changing the generation dispatch based on units physical ramping) are required in real time to mitigate

TABLE X
CONTROL STRATEGIES FOR DC TRANSMISSION SYSTEM IN CASE 3

Hr.	Rectifier				Inverter			
	First Control		Second Control		First Control		Second Control	
	Mode	Value	Mode	Value	Mode	Value	Mode	Value
1	CFA	5.00 Deg	CTR	1.09 pu	CC	129 A	CV	-1.10 pu
2	CFA	5.00 Deg	CTR	1.10 pu	CC	145 A	CV	-1.10 pu
3	CFA	5.00 Deg	CTR	1.05 pu	CC	121 A	CV	-1.06 pu
4	CFA	5.00 Deg	CTR	1.08 pu	CC	104 A	CV	-1.10 pu
5	CC	130 A	CV	1.08 pu	CAE	6.08 Deg	CP	-43.03MW
6	CFA	5.00 Deg	CTR	1.09 pu	CC	135 A	CV	-1.10 pu
7	CFA	5.00 Deg	CTR	1.09 pu	CC	129 A	CV	-1.10 pu
8	CFA	5.00 Deg	CTR	1.10 pu	CC	160 A	CV	-1.09 pu
9	CFA	5.00 Deg	CTR	1.10 pu	CC	164 A	CV	-1.09 pu
10	CFA	5.00 Deg	CTR	1.10 pu	CC	170 A	CV	-1.08 pu
11	CC	171 A	CP	58.48MW	CAE	5.39 Deg	CV	-1.08 pu
12	CFA	5.00 Deg	CTR	1.10 pu	CC	166 A	CV	-1.09 pu
13	CFA	5.00 Deg	CTR	1.10 pu	CC	162 A	CV	-1.09 pu
14	CFA	5.00 Deg	CTR	1.09 pu	CC	133 A	CV	-1.10 pu
15	CFA	5.00 Deg	CTR	1.10 pu	CC	170 A	CV	-1.08 pu
16	CFA	5.00 Deg	CTR	1.10 pu	CC	172 A	CV	-1.08 pu
17	CC	166 A	CP	57.20MW	CAE	5.28 Deg	CV	-1.09 pu
18	CC	171 A	CV	1.10 pu	CAE	5.39 Deg	CP	-57.52MW
19	CFA	5.00 Deg	CTR	1.10 pu	CC	176 A	CV	-1.08 pu
20	CFA	5.00 Deg	CTR	1.10 pu	CC	173 A	CV	-1.09 pu
21	CFA	5.00 Deg	CTR	1.10 pu	CC	165 A	CV	-1.09 pu
22	CFA	5.00 Deg	CTR	1.10 pu	CC	172 A	CV	-1.08 pu
23	CC	169 A	CV	1.10 pu	CAE	5.30 Deg	CP	-56.91MW
24	CFA	5.00 Deg	CTR	1.10 pu	CC	164 A	CV	-1.09 pu

TABLE XI
UC IN CASE 5

Unit	Hours (1-24)
1	1 1
2	1 1
3	1 1
4	1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
5	0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1

violations. The total operating cost is \$64 849.43 which is higher than that without the contingency (Case 1).

Case 5: If CTGL2-3 occurs in the AC/DC transmission system described in Case 2, the system security would not be satisfied based on the UC results in Table V. The bus mismatches at hours 3, 4, and 6 cannot be eliminated by only changing the generation dispatch (corrective actions). Consequently, three Benders cuts (42) are generated to change the UC results. In this case, generating unit 3 is committed (preventive actions) at hours 3,4 and 6 to satisfy the system security in the event of CTGL2-3. In Table XI, the UC changes in comparison to that in Case 3 are shown in bold. The total operating cost increases to \$64 269.91.

Case 6: In this case, we run the SCUC when the DC system is a fixed injection at buses 1 and 5 at all hours. We also assume that the power factor at rectifier and inverter terminals is 0.98 and the DC line has no losses. Fig. 6 shows the total operating cost when the DC transmission system is modeled with different injections between 30 MW and 55 MW. It is obvious that the proposed model would result

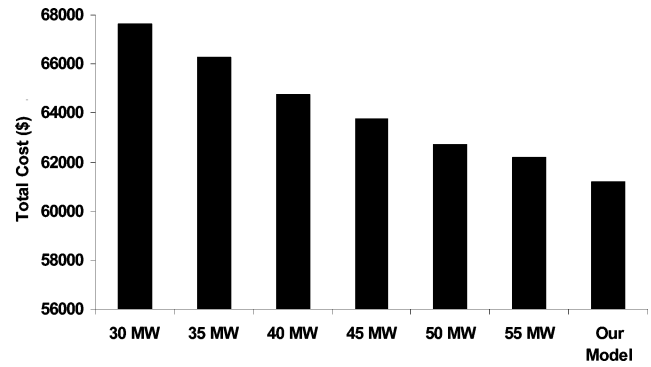


Fig. 6. Total generation costs for DC system with fixed injections.

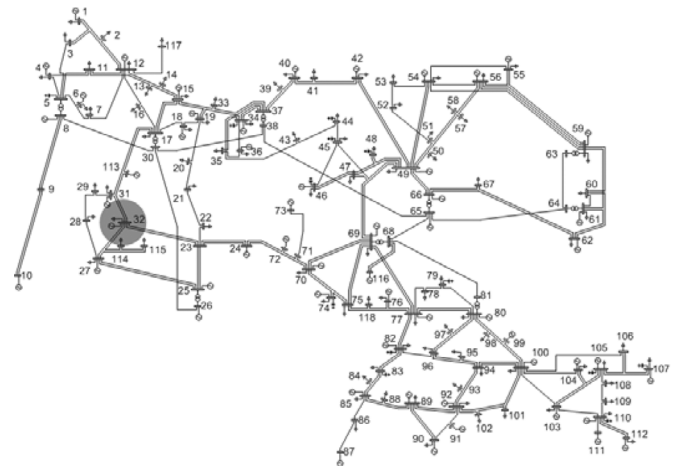


Fig. 7. IEEE 118-bus system.

in lower costs. There is no feasible SCUC solution when power flow injections are smaller than 30 MW or higher than 55 MW.

B. IEEE 118-Bus System

We consider a modified IEEE 118-bus system shown in Fig. 7 to study the proposed model. This system has 54 units, 186 branches, 14 capacitors, 9 tap-changing transformers, and 91 demand sides. The peak load of the system is 6000 MW that occurs at hour 21. The 118-bus system data are given at <http://motor.ece.iit.edu/DC/CSC/IEEE118.xls>.

The following cases are tested.

- Case 0:** Base case without transmission constraints
- Case 1:** SCUC solution with AC transmission constraints
- Case 2:** SCUC solution when the AC line 32–113 is replaced with a two-terminal DC transmission system. The rectifier terminal is connected to bus 113 and the inverter terminal to bus 32 with the same parameters as those in Table XVIII. This DC system is installed in the congested area based on SCUC results obtained in Case 1. The congested area is highlighted in Fig. 7.

The operating cost of UC in Case 0 is \$1 727 170 which increases to \$1 732 274 when the AC transmission constraints are considered in Case 1. The additional \$5103.64 cost is mainly

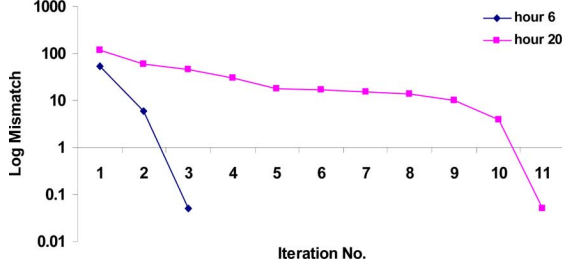


Fig. 8. Total hourly nodal mismatch in case 2.

due to the congestion of AC line 23–32 at peak hours which causes the commitment of expensive units. Case 2 replaces the AC line 32–113 with a two-terminal DC transmission line which results in an operating cost of \$1 729 950. The DC transmission flow has decreased the transmission congestion cost of AC line 23–32 from \$5103.64 to \$2780 when it transfers more power from bus 113 to bus 32. In Case 2, at the peak hour, the flow on DC transmission line 32–113 increases to 38.22 MW in comparison to 3 MW in Case 1.

Fig. 8 shows the convergence results for Case 2 at hours 6 and 20. Here additional iterations are required for SCUC to converge at higher load hours (11 iterations at hour 20) as compared to those at low-load hours (3 iterations at hour 6).

V. CONCLUSION

In this paper, controllable DC transmission systems are modeled in SCUC along with AC transmission systems. Benders decomposition is applied to effectively handle both AC and DC transmission constraints. The SCUC solution determines the optimal selection of DC control strategies and optimal hourly DC transmission schedule at the minimum cost while maintaining the system security. A set of optimal controls for DC transmission systems can successfully mitigate the AC transmission flow congestion and enhance power system economics. We compared the proposed model with a simplified DC transmission system model with fixed power flow injections/withdrawals that was widely used in SCUC. The simplified DC model may not provide a feasible SCUC solution while the proposed SCUC model guarantees the convergence. The test results also indicate that a given UC solution for an AC transmission system may not result in a feasible once the AC lines are partly replaced by DC lines. Additional adjustments in UC and ED, and transmission control are needed to ensure the feasibility.

APPENDIX A BASE FOR PER UNIT VALUES

The base values for AC/DC transmission systems are listed in Table XII.

TABLE XII
AC AND DC BASE VALUES

AC Base	DC Base
V_{base} (kV)	$V_{dc,base} = \frac{3\sqrt{2}}{\pi} V_{base}$ (kV)
S_{base} (MVA)	$P_{dc,base} = V_{dc,base} I_{dc,base} = S_{base}$ (MW)
$I_{base} = \frac{S_{base}}{\sqrt{3}V_{base}}$ (kA)	$I_{dc,base} = \frac{\pi}{\sqrt{6}} I_{base}$ (kA)
$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{V_{base}}{\sqrt{3}I_{base}}$ (Ω)	$Z_{dc,base} = \frac{V_{dc,base}}{I_{dc,base}} = \frac{18}{\pi^2} Z_{base}$ (Ω)
	$X_{c,base} = \frac{6}{\pi} Z_{base}$ (Ω)

APPENDIX B AC/CSC-DC JACOBIAN MATRICES

Reference [21] provides the Jacobian matrices $\mathbf{J1}$ and $\mathbf{J2}$ related to the AC system. The non-zero elements of Jacobian matrix $\mathbf{J1}$ related to the DC transmission system are

$$\begin{aligned} \frac{\partial \Delta P_m}{\partial V_{dc,h}} &= -I_{dc,h}, & \frac{\partial \Delta P_m}{\partial I_{dc,h}} &= -V_{dc,h}, \\ \frac{\partial \Delta Q_m}{\partial V_{dc,h}} &= -I_{dc,h} \tan \phi_h \\ \frac{\partial \Delta Q_m}{\partial I_{dc,h}} &= -V_{dc,h} \tan \phi_h, \\ \frac{\partial \Delta Q_m}{\partial \phi_h} &= -V_{dc,h} I_{dc,h} \left(\frac{1}{\cos^2 \phi_h} \right) \\ \frac{\partial \Delta R1_h}{\partial V_m} &= \begin{cases} -T_h \cos \alpha_h & \text{for rectifier} \\ +T_h \cos \alpha_h & \text{for inverter} \end{cases} \\ \frac{\partial \Delta R1_h}{\partial V_{dc,h}} &= 1, & \frac{\partial R1_h}{\partial I_{dc,h}} &= \begin{cases} -X_{c,h} & \text{for rectifier} \\ +X_{c,h} & \text{for inverter} \end{cases} \\ \frac{\partial \Delta R1_h}{\partial T_h} &= \begin{cases} +V_m \cos \alpha_h & \text{for rectifier} \\ -V_m \cos \alpha_h & \text{for inverter} \end{cases} \\ \frac{\partial \Delta R1_h}{\partial \alpha_h} &= \begin{cases} +T_h V_m \sin \alpha_h & \text{for rectifier} \\ -T_h V_m \sin \alpha_h & \text{for inverter} \end{cases} \\ \frac{\partial \Delta R2_h}{\partial V_m} &= \begin{cases} -0.995 T_h \cos \phi_h & \text{for rectifier} \\ +0.995 T_h \cos \phi_h & \text{for inverter} \end{cases}, & \frac{\partial \Delta R2_h}{\partial V_{dc,h}} &= 1 \\ \frac{\partial \Delta R2_h}{\partial T_h} &= \begin{cases} -0.995 V_m \cos \phi_h & \text{for rectifier} \\ +0.995 V_m \cos \phi_h & \text{for inverter} \end{cases} \\ \frac{\partial \Delta R2_h}{\partial \phi_h} &= \begin{cases} +0.995 T_h V_m \sin \phi_h & \text{for rectifier} \\ -0.995 T_h V_m \sin \phi_h & \text{for inverter.} \end{cases} \end{aligned}$$

The partial derivatives of $\Delta R3_h$ depend on the of the DC configuration. For example, the partial derivatives of $\Delta R3_h$ for the two-terminal DC system shown in Fig. 5 are

$$\begin{aligned} \frac{\partial \Delta R3_1}{\partial I_{dc,1}} &= 1, & \frac{\partial \Delta R3_1}{\partial V_{dc,1}} &= -\frac{1}{R_{dc}}, & \frac{\partial \Delta R3_1}{\partial V_{dc,2}} &= -\frac{1}{R_{dc}} \\ \frac{\partial \Delta R3_2}{\partial I_{dc,2}} &= 1, & \frac{\partial \Delta R3_2}{\partial V_{dc,1}} &= -\frac{1}{R_{dc}}, & \frac{\partial \Delta R3_2}{\partial V_{dc,2}} &= -\frac{1}{R_{dc}} \end{aligned}$$

where the subscribe 1 represents the rectifier, 2 for the inverter. Similarly, since the elements of Jacobian matrix **J3** depend on the DC configuration, we list non-zero elements of **J3** for the two-terminal DC transmission system shown in Fig. 5 as

$$\frac{\partial \Delta PL_{dc,1-2}}{\partial V_{dc,1}} = I_{dc,1}, \quad \frac{\partial \Delta PL_{dc,1-2}}{\partial I_{dc,1}} = V_{dc,1}$$

$$\frac{\partial \Delta PL_{dc,2-1}}{\partial V_{dc,2}} = -I_{dc,2}, \quad \frac{\partial \Delta PL_{dc,2-1}}{\partial I_{dc,2}} = -V_{dc,2}$$

where the subscribe 1 represents the rectifier, 2 for the inverter. The elements of Jacobian matrix **J4** are listed as

$$\frac{\partial \Delta P_{term,h}}{\partial V_{dc,h}} = I_{dc,h}, \quad \frac{\partial \Delta P_{term,h}}{\partial I_{dc,h}} = V_{dc,h}.$$

APPENDIX C
IEEE 14-BUS SYSTEM DATA

TABLE XIII
UNIT DATA

Unit	Bus No.	Bid (\$/MW)	Pmin (MW)	Pmax (MW)	Qmin (MVar)	Qmax (MVar)	Min On/Off (h)
1	1	8	50	150	-100	100	1
2	2	10	20	50	-50	50	1
3	3	18	12	80	-60	100	1
4	6	25	10	45	-30	30	1
5	8	40	10	45	-30	30	1

TABLE XIV
BUS DATA

Bus No.	Min Voltage (pu)	Max Voltage (pu)	Bus No.	Min Voltage (pu)	Max Voltage (pu)
1	1.04	1.07	8	1.02	1.10
2	1.03	1.05	9	1.00	1.03
3	1.00	1.02	10	1.00	1.05
4	1.00	1.03	11	0.98	1.05
5	1.00	1.04	12	1.04	1.07
6	1.05	1.10	13	1.01	1.07
7	1.01	1.05	14	0.98	1.07

TABLE XV
AC BRANCH DATA

Line No.	From Bus	To Bus	R (pu)	X (pu)	Charging Shunt (pu)	Flow Limit (MW)
1	1	2	0.0194	0.0592	0.0264	50
2	1	5	0.0540	0.2230	0.0246	65
3	2	3	0.0470	0.1980	0.0219	60
4	2	4	0.0581	0.1763	0.0187	60
5	2	5	0.0570	0.1739	0.0170	60
6	3	4	0.0670	0.1710	0.0173	60
7	4	5	0.0134	0.0421	0.0064	40
8	4	7	0.0000	0.2091	0.0000	65
9	4	9	0.0000	0.5562	0.0000	40
10	5	6	0.0000	0.2520	0.0000	65
11	6	11	0.0950	0.1989	0.0000	50
12	6	12	0.1229	0.1558	0.0000	50
13	6	13	0.0662	0.1303	0.0000	50
14	7	8	0.0000	0.1762	0.0000	50
15	7	9	0.0000	0.1100	0.0000	30
16	9	10	0.0318	0.0845	0.0000	50
17	9	14	0.1271	0.2704	0.0000	50
18	10	11	0.0821	0.1921	0.0000	50
19	12	13	0.2209	0.1999	0.0000	50
20	13	14	0.1709	0.3480	0.0000	50

TABLE XVI
DISTRIBUTION FACTORS OF LOADS AT DIFFERENT BUSES

Load	At Bus	MW Load Factor	MVar Load Factor
1	2	0.0838	0.1728
2	3	0.3637	0.2585
3	4	0.1846	-0.0531
4	5	0.0293	0.0218
5	6	0.0432	0.1020
6	9	0.1139	0.2259
7	10	0.0347	0.0789
8	11	0.0135	0.0245
9	12	0.0236	0.0218
10	13	0.0521	0.0789
11	14	0.0575	0.0680

TABLE XVII
HOURLY LOAD DISTRIBUTION

Hour	MW	MVar	Hour	MW	MVar
1	181.30	51.45	13	207.20	58.80
2	170.94	48.51	14	196.84	55.86
3	150.22	42.63	15	227.92	64.68
4	103.60	29.40	16	233.10	66.15
5	129.50	36.75	17	220.15	62.48
6	155.40	44.10	18	230.51	65.42
7	181.30	51.45	19	243.46	69.09
8	202.02	57.33	20	253.82	72.03
9	212.38	60.27	21	259.00	73.50
10	227.92	64.68	22	233.10	66.15
11	230.51	65.42	23	225.33	63.95
12	217.56	61.74	24	212.38	60.27

TABLE XVIII
CONVERTER DATA FOR THE DC TRANSMISSION SYSTEM

Type	X_c (pu)	α_{min} (Deg)	α_{max} (Deg)	$V_{dc,min}$ (pu)	$V_{dc,max}$ (pu)	Min Tap	Max Tap
Rec.	0.126	5	30	0.95	1.20	0.9	1.1
Inv.	0.175	5	30	0.90	1.10	0.9	1.5

TABLE XIX
DC BUS DATA FOR THE DC TRANSMISSION SYSTEM

Bus No.	Min Voltage (pu)	Max Voltage (pu)
1	0.95	1.20
2	0.90	1.10

TABLE XX
DC BRANCH DATA

Line No.	From DC Bus	To DC Bus	R (pu)	Flow Limit (MW)
1	1	5	0.0342	65

REFERENCES

[1] K. R. Padiyar, *HVDC Power Transmissions*. New York: Wiley, 1990.
 [2] A. E. Hammad and W. F. Long, "Performance and economic comparisons between point-to-point HVDC transmission and hybrid back-to-back HVDC/AC transmission," *IEEE Trans. Power Del.*, vol. 5, no. 2, pp. 1137–1144, Apr. 1990.
 [3] S. Wang, J. Zhu, L. Trinh, and J. Pan, "Economic assessment of HVDC project in deregulated energy markets," in *Proc. 3rd Int. Conf. Electric Utility Deregulation and Restructuring and Power Technologies*, Nanjing, China, Apr. 2008.
 [4] X. P. Zhang and L. Yao, "A vision of electricity network congestion management with FACTS and HVDC," in *Proc. 3rd Int. Conf. Electric Utility Deregulation and Restructuring and Power Technologies*, Nanjing, China, Apr. 2008.

- [5] E. W. Kimbark, *Direct Current Transmission*. New York: Wiley, 1971.
- [6] M. Ishikawa, S. Horiuchi, S. Hirose, T. Sakai, and T. Horiuchi, "HVDC transmission control equipment with high reliability," *IEEE Trans. Power Del.*, vol. 1, no. 2, pp. 254–263, Apr. 1986.
- [7] P. Kundur, *Power System Stability and Control*. New York: McGraw-Hill, 1994.
- [8] J. Arrillaga, Y. H. Liu, and N. R. Watson, *Flexible Power Transmission: The HVDC Options*. New York: Wiley, 2007.
- [9] C. N. Lu, S. S. Chen, and C. M. Ong, "The incorporation of HVDC equations in optimal power flow methods using sequential quadratic programming technique," *IEEE Trans. Power Syst.*, vol. 3, no. 3, pp. 1005–1011, Aug. 1988.
- [10] H. Ambriz-Perez, E. Acha, and C. R. Fuerte-Esquivel, "High voltage direct current modelling in optimal power flows," *Elect. Power Energy Syst.*, no. 30, pp. 157–168, 2008.
- [11] M. M. El-Marsafawy and R. M. Mathur, "A new, fast technique for load flow solution of integrated multi-terminal DC/AC systems," *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 1, pp. 246–255, Jan./Feb. 1980.
- [12] J. Reeve, G. Fahny, and B. Stott, "Versatile load flow method for multi-terminal HVDC systems," *IEEE Trans. Power App. Syst.*, vol. PAS-96, no. 3, pp. 925–933, May/June 1977.
- [13] T. Smed, G. Anderson, G. B. Sheble, and L. L. Grigsby, "A new approach to AC/DC power flow," *IEEE Trans. Power Syst.*, vol. 6, no. 3, pp. 1238–1244, Aug. 1991.
- [14] V. Sarkar and S. A. Khaparde, "Implementation of LMP-FTR mechanism in an AC-DC system," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 737–746, May 2008.
- [15] Z. Li and M. Shahidehpour, "Security-constrained unit commitment for simultaneous clearing of energy and ancillary services markets," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1079–1088, May 2005.
- [16] S. Harvey, Internal NYISO HVDC Controllable Line Scheduling: Concept of Operation, May 2004. [Online]. Available: <http://www.nyiso.com>.
- [17] N. Stretch, M. Kazerani, and R. E. ShatShat, "A current-sourced converter based HVDC light transmission system," in *Proc. IEEE Int. Symp. Industrial Electronics*, Jul. 2006, vol. 3.
- [18] A. Pizano-Martinze, C. R. Fuerte-Esquivel, H. Ambriz-Perez, and E. Acha, "Modeling of VSC-based DC systems for a Newton-Raphson OPF algorithm," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1794–1803, Nov. 2007.
- [19] X. P. Zhang, "Multiterminal voltage-sourced converter-based DC models for power flow analysis," *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 1877–1884, Nov. 2004.
- [20] X. Wei, J. H. Chow, B. Fardanesh, and A. A. Edris, "A common modeling framework of voltage-sourced converters for load flow, sensitivity, and dispatch analysis," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 934–941, May 2004.
- [21] Y. Fu, M. Shahidehpour, and Z. Li, "AC contingency dispatch based on security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 21, no. 2, pp. 897–908, May 2006.
- [22] D. A. Braunagel, L. A. Kraft, and J. L. Whyson, "Inclusion of DC converter and transmission equations directly in a Newton power flow," *IEEE Trans. Power App. Syst.*, vol. PAS-95, no. 1, pp. 76–88, 1976.
- [23] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems*. New York: Wiley, 2002.
- [24] Y. Fu and M. Shahidehpour, "Fast SCUC for large-scale power systems," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 2144–2151, Nov. 2007.
- [25] Y. Fu, M. Shahidehpour, and Z. Li, "Security constrained unit commitment with AC constraints," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1001–1013, May 2005.

Azim Lotfjou (M'09) received the B.S. and M.S. degrees in electrical engineering from Sharif University of Technology, Tehran, Iran, in 2001 and 2004, respectively. Presently, he is pursuing the Ph.D. degree at the Illinois Institute of Technology, Chicago.

He is a Research Assistant at the Illinois Institute of Technology.

Mohammad Shahidehpour (F'01) is a Carl Bodine Professor in the Electrical and Computer Engineering Department at Illinois Institute of Technology (IIT), Chicago.

Dr. Shahidehpour is Vice President of Publications for the IEEE Power and Energy Society.

Yong Fu (M'05) received the B.S. and M.S. degrees in electrical engineering from Shanghai Jiaotong University, Shanghai, China, in 1997 and 2002, respectively, and the Ph.D. degree in electrical engineering from the Illinois Institute of Technology, Chicago, in 2006.

Zuyi Li (M'01) is an Associate Professor in the Electrical and Computer Engineering Department at Illinois Institute of Technology, Chicago.