Extended Benders Decomposition for Two-Stage SCUC

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***Abstract*—This letter presents the solution of a two-stage security-constrained unit commitment (SCUC) problem. The proposed SCUC model could include integer variables at the second stage. A framework of extended Benders decomposition with linear feasibility and optimality cuts is proposed for the solution of mixed-integer programming (MIP) problems at both stages. Test results show the effectiveness of the proposed methodology.**

***Index Terms*—L-shaped decomposition, MIP, SCUC.**

NOMENCLATURE

$\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{E}, \boldsymbol{F}, \boldsymbol{g}$	Coefficients vector, matrix of MIP problem.
$\mathbf{e}, \mathbf{I}$	Vector of ones and identity matrix.
i, j, k, s	Index of nodes in branch and bound trees, iterations, and scenarios.
m_1, n_1, m_2^s, n_2^s	Dimension of first/second stage problems.
$O(\,), IF(\,), Q(\,)$	Dual price function of MIP problem.
$OR(\,), IFR(\,)$	Dual price function of LP-relaxation of MIP.
S, p^s	Number of scenarios and their probabilities.
$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{t}$	First stage, second stage, and slack variables.
$\mathbf{X}, \mathbf{Y}^s$	Spanning space of x and y^s variables.
z, θ	Variables represent objective function values.
δ	Ancillary binary variable.
μ, π	Dual variables.
ε, M	Predefined tolerance and a large number.

I. PROPOSED MIP PROBLEM

THE security-constrained unit commitment (SCUC) problem shown in Table I is solved in two stages according to its specific structure. Generally, the first stage corresponds to an optimal decision, while the second stage would examine the viability and optimality of first stage decisions.

Benders cut applications are based on a prerequisite that the subproblem corresponding to the second stage is LP or convex NLP [1]. In practice, variables in the second stage can be integer or semi-continuous. Consider the following examples:

1) integer variables in power transmission control;
2) status of quick start units in post-contingency evaluation.

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TABLE I
SCUC DECOMPOSITION STRATEGIES

Problem	First Stage	Second Stage
UC with network constraints	UC	Transmission constraints
Contingency-based SCUC	UC with pre-contingency transmission constraints	Post-contingency transmission constraints
Two-stage stochastic SCUC	Decision costs and constraints	Recourse costs and constraints

We have developed an extended Benders decomposition method which can be applied to such SCUC problems. The SCUC problem is represented as a MIP formulation (1)–(4). Decision variables x at the first stage and y at the second stage in P could be either integer or continuous. The constraint structure is L-shaped and no coupling among subproblems:

$$\mathrm{P}: \quad Min\; z = \left\{ \boldsymbol{c} \cdot \boldsymbol{x} + \sum_s p^s \cdot \boldsymbol{d}^s \cdot \boldsymbol{y}^s \right\} \tag{1}$$

$$s.t. \quad \boldsymbol{A} \cdot \boldsymbol{x} \geq \boldsymbol{b} \tag{2}$$

$$\boldsymbol{E}^s \cdot \boldsymbol{x} + \boldsymbol{F}^s \cdot \boldsymbol{y}^s \geq \boldsymbol{g}^s \quad \forall s \in \{1, 2, \cdots, S\} \tag{3}$$

$$\boldsymbol{x} \in \mathbf{X} = span\left\{\boldsymbol{R}^{m_1} \times \{0,1\}^{n_1}\right\}, \quad \boldsymbol{y}^s \in \mathbf{Y}^s = span\left\{\boldsymbol{R}^{m_2^s} \times \{0,1\}^{n_2^s}\right\}. \tag{4}$$

Nonlinear feasibility and optimality cuts were developed in [3] via the general duality theory and showed that an L-shaped decomposition can converge within a finite time. The formulation of optimality and feasibility cuts are given in (12) and (13). After introducing integer variables, the linear feasibility and optimality cuts (21)–(23) would transform the two-stage SCUC problem into a MIP set, which can be incorporated into the master problem and lead to the global optimal solution.

As to the extended Benders decomposition, the mixed-integer master problem (BD-MP) and subproblems (BD-SP) are given in (5)–(8) and (9), respectively:

$$\mathrm{BD-MP}: \quad z_{LB} = \underset{x, \theta^s}{Min} \left\{ \boldsymbol{c} \cdot \boldsymbol{x} + \sum_s p^s \cdot \theta^s \right\} \tag{5}$$

$$s.t. \quad \boldsymbol{A} \cdot \boldsymbol{x} \geq \boldsymbol{b} \quad \boldsymbol{x} \in \mathbf{X},\ \theta^s \in \boldsymbol{R}^1 \tag{6}$$

$$O^s(\boldsymbol{x}) - \theta^s \leq 0 \tag{7}$$

$$IF^s(\boldsymbol{x}) \leq 0 \tag{8}$$

$$\mathrm{BD-SP}: \quad \underset{y^s}{Min} \left\{ \boldsymbol{d}^s \cdot \boldsymbol{y}^s | \boldsymbol{F}^s \cdot \boldsymbol{y}^s \geq \boldsymbol{g}^s - \boldsymbol{E}^s \cdot \hat{\boldsymbol{x}}, \boldsymbol{y}^s \in \mathbf{Y}^s \right\}. \tag{9}$$

Here, $\hat{\boldsymbol{x}}$ is the current BD-MP solution. In addition, (7) and (8) represent optimality and feasibility cuts, respectively, which are formed by solving BD-SP. The BD-MP solution provides a lower bound while a feasible BD-SP solution leads to an upper bound. The optimal solution of P is calculated when upper and lower bounds are sufficiently close according to (10):

$$(z_{UB} - z_{LB})/z_{UB} \leq \varepsilon. \tag{10}$$

We form feasibility and optimality cuts by the dual price function when BD-SP is a MIP problem. BD-SP is nonconvex so its dual problem (BD-DSP) is given in (11) which is based on the general duality theory:

$$\underset{Q^s()}{Max} \left\{ Q^s(\boldsymbol{g}^s - \boldsymbol{E}^s \cdot \hat{\boldsymbol{x}}) | Q^s(\boldsymbol{F}^s \cdot \boldsymbol{y}^s) \leq \boldsymbol{d}^s \cdot \boldsymbol{y}^s, \forall \boldsymbol{y}^s \in \mathbf{Y}^s \right\}. \tag{11}$$

This problem is considered as finding MIP sensitivities of (9) corresponding to the right-hand side of $(\boldsymbol{g}^s - \boldsymbol{E}^s \cdot \boldsymbol{x})$. According to [2], a lower bound at a non-terminal node, which is based on branch and bound, is proposed in (12):

$$O_i^s(\boldsymbol{x}) = Max \left\{ OR_i^s(\boldsymbol{x}), Min \left\{ O_{i,LB}^s(\boldsymbol{x}), O_{i,UB}^s(\boldsymbol{x}) \right\} \right\} \tag{12}$$

where $OR_i^s(\boldsymbol{x})$ is the dual price function of the LP-relaxation of MIP (9) at the current node i. $O_{i,LB}^s(\boldsymbol{x})$, $O_{i,UB}^s(\boldsymbol{x})$ are dual price functions of lower and upper partition nodes of current i. If the relaxed LP at current node is infeasible, $OR_i^s(\boldsymbol{x}) = +\infty$ is set and the feasibility cut (13) is added:

$$IFR_i^s(\boldsymbol{x}) \leq 0. \tag{13}$$

If the current node is a root node of branch and bound tree, the optimality cuts and feasibility cuts based on (12) and (13) are fed back to BD-MP. Reference [2] offers another dual price function (14) where node i belongs to the set of feasible terminal nodes in the tree generated by branch and bound. Coefficients in (15) are determined by solving the LP problem with the primal-dual method (16)–(18):

$$O^s(\boldsymbol{x}) = \underset{i}{Min} \left\{ OR_i^s(\boldsymbol{x}) | \, i \in \text{set of feasible terminal nodes} \right\} \tag{14}$$

$$\begin{aligned} & OR_i^s(\boldsymbol{x}) = \boldsymbol{\mu}_i^s \cdot (\boldsymbol{g}^s - \boldsymbol{E}^s \cdot \boldsymbol{x}) + \overline{\boldsymbol{\pi}}_i^s \cdot \overline{\boldsymbol{y}}^s + \underline{\boldsymbol{\pi}}_i^s \cdot \underline{\boldsymbol{y}}^s && (15)\\ & Min\, \boldsymbol{d}^s \cdot \boldsymbol{y}^s && (16)\\ & s.t. \quad \boldsymbol{F}^s \cdot \boldsymbol{y}^s \geq \boldsymbol{g}^s - \boldsymbol{E}^s \cdot \hat{\boldsymbol{x}} \qquad \boldsymbol{\mu}_i^s \geq 0 && (17)\\ & \qquad \overline{\boldsymbol{y}}_i^s \geq \boldsymbol{y}^s \geq \underline{\boldsymbol{y}}_i^s \quad \overline{\boldsymbol{\pi}}_i^s \leq 0, \underline{\boldsymbol{\pi}}_i^s \geq 0. && (18) \end{aligned}$$

If a terminal node j of a BD-SP scenario is infeasible, we add a slack vector to the LP (19) corresponding to the terminal node. We form feasibility cuts (20) the same way as optimality cuts by finding the dual price function of (19). Compared to (12), the combination of (14) and (20) is a weaker bound which leads to a global optimal solution within a finite time [3]:

$$\underset{\boldsymbol{t}^s, \boldsymbol{y}^s}{Min} \left\{ \boldsymbol{e}^s \cdot \boldsymbol{t}^s | \boldsymbol{F}^s \cdot \boldsymbol{y}^s + \boldsymbol{I}^s \cdot \boldsymbol{t}^s \geq \boldsymbol{g}^s - \boldsymbol{E}^s \cdot \hat{\boldsymbol{x}}, \; \overline{\boldsymbol{y}}_j^s \geq \boldsymbol{y}^s \geq \underline{\boldsymbol{y}}_j^s \right\} \tag{19}$$

$$IF^s(\boldsymbol{x}) = \underset{j}{Min} \left\{ IFR_j^s(\boldsymbol{x}) | j \in set \text{ of infeasible terminal nodes} \right\}. \tag{20}$$

Substituting (14) and (20) into (7) and (8), respectively, the BD-MP becomes a mixed-integer bilevel optimization problem.

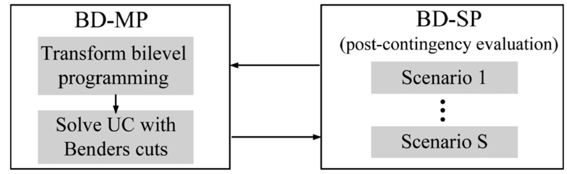

Fig. 1. Decomposition framework.

TABLE II
SECOND-STAGE STOCHASTIC UC SOLUTION (MW)

Scenario	Probability	Period	Load	Unit 1	Unit 2	Unit 3
1	0.5	T=1	90	90	0	0
		T=2	110	100	10	0
2	0.5	T=1	95	95	0	0
		T=2	100	100	0	0

We introduce additional integer variables δ in the BD-SP so that (21)–(23) can be used instead of (7) and (8):

$$OR_i^s(\boldsymbol{x}) - \theta^s - (1 - \delta_i^s) \cdot M \leq 0 \quad i \in \text{set of feasible terminal nodes} \tag{21}$$

$$IFR_j^s(\boldsymbol{x}) - \left(1 - \delta_j^s\right) \cdot M \leq 0 \quad j \in \text{set of infeasible terminal nodes} \tag{22}$$

$$\sum_i \delta_i^s + \sum_j \delta_j^s = 1 \quad \delta_i^s, \delta_j^s \in \{0, 1\}. \tag{23}$$

The decomposition solution in Fig. 1 is summarized as follows:

Step 0) Set the lower bound $z_{LB} = 0$, $z_{UB} = +\infty$. Initialize the iteration counter of outer loop $k = 1$ and tolerance ε.

Step 1) Introduce (21)–(23) instead of (7) and (8) for Benders cuts and solve the MIP-based BD-MP.

Step 2) Go to Step 2.1 if BD-SPs are feasible for all scenarios, based on the solution of BD-MP. Otherwise, go to Step 2.2.

Step 2.1) For each scenario, form an optimality cut (7) by branch and bound. Update z_{UB}. If (10) is satisfied, stop the procedure. Else, go to Step 1 and set $k = k+1$.

Step 2.2) For infeasible scenarios, use branch and bound to form feasibility cuts (8). Go to Step 1 and set $k = k + 1$.

II. NUMERICAL EXAMPLE

Three generators are scheduled by a two-stage stochastic UC to supply a load with uncertainty levels shown in Table II. Unit 1 is a coal unit without a quick start capacity; thus, its commitment decision can only be made at the first stage. However, units 2 and 3 have quick start capability and their commitment can be adjusted quickly at the second stage. The second stage is comprised of two parallel MIP subproblems corresponding to two scenarios with commitment decisions for units 2 and 3. The problem is solved by applying the proposed extended Benders decomposition. Table II shows the second-stage solution of the stochastic UC by the proposed method, which is the same as that of solving the MIP problem without decomposition by applying CPLEX. The first stage solution shows that unit 1 is committed in both scenarios. The second stage solution shows that unit 2 is committed only at hour 2 in scenario 1. Two iterations between BD-MP and BD-SP are executed, and two feasibility cuts (21) and two optimality cuts (22) are generated before (10) is satisfied. More information on this example is presented in http://motor.ece.iit.edu/data/bds_example.doc.

III. Discussions and Summary

The SCUC problem is solved by introducing the decomposition presented in Table I for practical systems with a large number of scenarios/contingencies. The deterministic SCUC problems in Table I, which require no optimality cuts, can be solved by a limited number of feasibility cuts. With N1 quick-start units, S contingencies, and N2 discrete network equipment, the number of the second-stage integer variables is $\mathrm{S}^*(\mathrm{N1}+\mathrm{N2})$, which could be a large number and make it impossible to solve the problem without decomposition. In the proposed method, the number of feasibility cuts (22) is much less than $\mathrm{S}^*(\mathrm{N1}+\mathrm{N2})$ which can be solved easily for large systems. The stochastic SCUC in Table I, which considers recourse costs, would require optimality and feasibility cuts formed in every scenario. For a global optimization, at most $2^{(N1+N2)\cdot S}$ integer variables would be introduced into the master problem as shown in (21)–(23). The type 1 Special Ordered Set (23) facilitates the branching process of branch-and-bound method for a faster convergence. The potential future studies could seek better methodologies for implementing (12), (13), (14), and (20) such that a few terminal nodes in the tree can be excluded in (14) and (20). Heuristic algorithms may be embedded into the proposed framework to provide a tradeoff between the computing time and the accuracy.

References

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