# Transmission Switching in Security-Constrained Unit Commitment

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Abstract—Transmission switching (TS) is introduced in security-constrained unit commitment (SCUC) for alleviating transmission violations and reducing operating costs. The SCUC problem is decomposed into the unit commitment (UC) master problem and the TS subproblem. The UC master problem finds the optimal hourly schedule of generating units. The TS subproblem uses this solution for transmission switching to find the optimal dispatch of units when considering network constraints. The TS subproblem also examines contingencies and identifies required changes to the UC master problem solution when contingencies cannot be mitigated in the TS subproblem. To propose a practical TS model, the standing phase angle difference limit is considered and relevant constraints are added to the TS subproblem. The case studies exhibit the effectiveness of the proposed approach.

*Index Terms*—Benders decomposition, mixed-integer programming (MIP), security-constrained unit commitment, transmission switching.

# NOMENCLATURE

1) Indices:

b,m,n	Index for bus .
c	Index for contingency.
i	Index for unit.
l	Index for line.
ns	Index for nonswitchable lines.
s	Index for switchable lines.
t	Index for time.
$\wedge$	Index for given variables.
2) Sets:	
$L_b$	Set of lines connected to bus b.
$U_b$	Set of units connected to bus b.
3) Parar	neters:
$F_{ci}$	Production cost function of unit $i$ .
NB	Number of buses.
NC	Number of contingencies.
NL	Number of lines.

NG	Number of units.										
NS	Number of switchable lines.										
NNS	Number of nonswitchable lines.										
NT	Number of time periods.										
$D_t$	System demand at time t.										
$Pd_{bt}$	Load at bus $b$ at time $t$ .										
$P_{i,\min}$	Minimum power generation of unit <i>i</i> .										
$P_{i,\max}$	Maximum power generation of unit <i>i</i> .										
$PL_{l,\max}$	Maximum capacity of line <i>l</i> .										
$UX_{it}^c$	State of unit $i$ at time $t$ in contingency $c$ .										
$UY_{lt}^c$	State of line $l$ at time $t$ in contingency $c$ .										
$x_l$	Reactance of line <i>l</i> .										
$\Delta_l$	Maximum standing phase angle difference of line $l$ .										
4) Varial	ples:										
$I_{it}$	Commitment state of unit $i$ at time $t$ .										
$P_{it}$	Generation of unit $i$ at time $t$ .										
$PL_{lt}^{ns}$	Power flow of nonswitchable line $l$ at time $t$ .										
$PL_{lt}^s$	Power flow of switchable line $l$ at time $t$										
$SL_{bt,1}, SI$	$L_{bt,2}$ Slack variables for power mismatch at bus <i>b</i> at time <i>t</i> .										
$v_t$	Power mismatch at time t.										
$w_t^c$	Power mismatch at time $t$ and contingency $c$ .										
$z_{lt}$	Switching state of line $l$ at time $t$ .										
$\pi_{it}$	Marginal change in violations with increase in unit $i$ generation at time $t$ .										
$\delta_{mt}$	Phase angle of bus $m$ at time $t$ .										

# I. INTRODUCTION

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**O** NE of the approaches for mitigating transmission flow violations is to switch power system elements. Corrective transmission switching (TS) can provide economic benefits when compared with other control methods such as generation unit rescheduling or load shedding [1], [2]. In [3], TS was considered by applying current sources at certain bus terminals in



Fig. 1. SCUC using TS.

the base network. The injected currents were represented as control variables in the optimal TS subproblem. In [4], TS was employed as a corrective action to mitigate contingencies. Also, TS was used to model outages in optimal power flow. In [5], a TS method was devised for calculating N - 1 secure states. A linear switching model was applied to model control actions applied to contingency constraints. In [6], TS was considered as a means of mitigating violations in transmission flows and bus voltages. Also, practical issues related to switching operations were addressed. In these papers, transformer tap adjustments and static VAR compensators were recognized as a means of controlling the voltage problems [6]. In [7], a TS algorithm was developed for switching off lines and buses to mitigate contingencies. This TS algorithm was based on a sparse inverse technique and fast decoupled power flow. Also in [8], TS provided system operators with a congestion management tool.

These works showed that TS provides flexible control actions for voltage stability, congestion management, loss reduction, and system security. However, TS can provide economic benefits, where this concept was first introduced in a pioneering work by O'Neill et al. [9] in a market context. Moreover in [10] and [11], the problem of finding an optimal generation dispatch and transmission topology was investigated. In [10], the TS problem was solved only for the base case dispatch of the system, while in [11], the N – 1 contingency criterion was considered as well. A mixed-integer programming (MIP) model was used, which employed binary variables to represent the transmission line state. These papers found that TS could achieve large improvements in dispatch costs. The work in [12] was an extension of [10] which demonstrated that market participants are subject to system uncertainties when considering TS. The paper discussed ways that topology changes could affect nodal prices, load payments, generation revenues, congestion costs, and flowgate prices.

In this paper, the optimal TS for alleviating overloads is considered in SCUC that would take into account prevailing generating unit and transmission network constraints. Fig. 1 depicts the hierarchy for calculating SCUC by applying TS. The large-scale SCUC solution can represent a computation burden in power system operations. Since it is generally viewed as impractical to solve the generation unit and the entire set of network constraints together in SCUC, various decomposition approaches are considered by commercial SCUC packages.

The proposed decomposition applied to the SCUC problem in this paper consists of the UC master problem and the TS subproblem. Accordingly, Benders decomposition is utilized to decompose the SCUC problem into smaller and easier to solve subproblems. Benders decomposition is mathematically sound and can easily be applied to large-scale systems. The optimality of Benders decomposition as well as its applicability to power system problems in practical cases are discussed in [13]–[17].

The UC master problem in Fig. 1 will find the optimal schedule of units, considering the prevailing UC constraints. The initial optimal schedule of generating units is obtained based on the available market data. The UC solution is used in the TS subproblem to find the optimal hourly dispatch of units, considering transmission constraints and the switching capability of transmission lines.

The TS subproblem in Fig. 1 consists of three main blocks as shown in Fig. 2. The TS Feasibility Check inspects the UC result to find whether a feasible TS solution can be found in the base case (without considering contingencies). If violations persist, Benders cuts are generated and added to the UC master problem. After satisfying the TS feasibility, the Optimal TS Scheduling block will utilize the UC solution to find the optimal dispatch of generating units and the state of switchable lines in the base case and contingencies. The Transmission Contingencies Check block will then examine the Optimal TS Scheduling results as to whether a converged dc power flow solution can be obtained. The Transmission Contingencies Check block will also adjust the transmission flows to mitigate any existing bus mismatches. In the case of violations, Benders cuts are formed and added to the Optimal TS Scheduling block for the redispatch of generating units.

If a converged dc power flow solution can be found for a contingency considered in the TS subproblem, this contingency will be labeled as controllable. However, if a converged dc power flow solution does not exist after a certain number of iterations between the Optimal TS Scheduling and the Transmission Contingencies Check blocks, i.e., the violations still persist in the Transmission Contingencies Check block, the considered contingency will be labeled as uncontrollable. The controllable contingencies would not require any further processing as they can be handled by the existing UC solution. The uncontrollable contingencies, which cannot be handled in the TS subproblem, are sent back to the UC master problem to find a new (preventive) UC schedule. By labeling the contingencies, the controllable ones are considered first (corrective) and then uncontrollable ones are handled (preventive). In other words, the proposed approach examines less expensive corrective actions before resorting to the more expensive preventive actions.

One of the concerns in performing the TS in successive hours is the possibility of excessive standing phase angle difference across a switched line. If a line closing causes a loop closure, rapid changes may be required in the generation dispatch, which could otherwise impact the generator shaft. The corresponding torque induced in the rotor of a generator could cause generator fatigue and equipment failure [18]–[20]. This may happen to any generators irrespective of their connectivity to the closed line. So, it is required to limit the standing phase angle difference to safeguard the rotor shaft. Usually a generation redispatch upon line closures would bring the standing phase angles back to the safe region. Different approaches are proposed to obtain the minimum generation redispatch needed to achieve



Fig. 2. Flowchart of SCUC using TS.

desired standing phase angles in a reasonable time [21]–[25]. This problem which is inevitable in restoration practices may also occur in the normal operation of power systems when attempting to reclose a single line that is a part of a transmission loop [20]. To present a practical TS model, the standing phase angle difference limit of switchable lines is formulated in our proposed model.

The rest of the paper is organized as follows. Section II presents the new proposed approach and formulates different parts of it. Section III conducts the numerical simulations and in detail discusses a six-bus system and the IEEE 118-bus system. Finally, concluding remarks are discussed in Section IV.

#### **II. SCUC PROBLEM FORMULATION**

The SCUC problem depicted in Fig. 2 is formulated as follows.

#### A. UC Master Problem (Optimal Hourly Schedule of Units)

The objective of the UC master problem is to determine the day-ahead schedule of generating units in order to minimize the system operating cost while satisfying the prevailing constraints. A complete list of UC constraints and optimization techniques is found in [13].

## B. TS Subproblem

The TS subproblem consists of three parts, TS Feasibility Check, Optimal TS Scheduling, and Transmission Check as discussed below.

1) TS Feasibility Check: TS Feasibility Check which is a MIP problem would examine the possibility of a feasible TS solution. The objective is

$$Min \sum_{t=1}^{NT} \sum_{b=1}^{NB} (SL_{bt,1} + SL_{bt,2}).$$
(1)

The bus power mismatch is presented by (2), where  $SL_{bt,1}$ and  $SL_{bt,2}$  are surplus and deficit variables. The other constraints are power flow of switchable lines (3)–(6), power flow of nonswitchable lines (7)–(9), standing phase angle difference limits (10)–(11), and non-islanding constraint (12):

$$\sum_{i \in U_b} P_{it} - \sum_{l \in L_b} PL_{lt}^s - \sum_{l \in L_b} PL_{lt}^{ns} - Pd_{bt} + SL_{bt,1} - SL_{bt,2} = 0$$
(2)  
(b = 1,...,NB)(s = 1,...,NS)(ns = 1,...,NNS)  
$$PL_{lt}^s - (\delta_{mt} - \delta_{nt})/x_l + M(1 - z_{lt}) \ge 0$$
(3)  
(t = 1,...,NT)(l = 1,...,NL)(s = 1,...,NS)

$$PL_{lt}^{s} - (\delta_{mt} - \delta_{nt})/x_{l} - M(1 - z_{lt}) \le 0$$
(4)

$$(t = 1, \dots, NT)(l = 1, \dots, NL)(s = 1, \dots, NS)$$

$$(5)$$

$$-PL_{lt}^{s} \leq PL_{l,\max}z_{lt}$$
(6)

$$(t = 1, \dots, NT)(l = 1, \dots, NL)(s = 1, \dots, NS)$$

$$JL_{lt}^{ns} = (\delta_{mt} - \delta_{nt})/x_l \tag{7}$$

$$(t = 1, \dots, NT)(l = 1, \dots, NL)(ns = 1, \dots, NNS)$$

$$L_{lt} \ge I L_{l,\max} \tag{8}$$

$$(t = 1, \dots, NL)(l = 1, \dots, NL)(ns = 1, \dots, NNS)$$
$$-PL_{lt}^{ns} \le PL_{l,\max}$$
(9)

$$(t = 1, \dots, NT)(l = 1, \dots, NL)(ns = 1, \dots, NNS)$$

$$(t = 1, \dots, NT)(l = 1, \dots, NL)$$

$$(10)$$

$$\delta_{mt} - \delta_{nt} \ge -\Delta_l - M z_{lt} - M(1 - z_{l(t-1)}) \tag{11}$$

$$(t=1,\ldots,NT)(l=1,\ldots,NL)$$

$$\sum_{l \in L_b} z_{lt} \ge 1 \qquad (t = 1, \dots, NT) \tag{12}$$

where M is a large positive number.

The TS constraints use a single binary variable  $z_{lt}$ . When this variable is equal to one, power flow constraints on switchable lines will be the same as those of other lines. In such a case, the line will be treated the same as other lines. When the line's binary variable is zero, (3)–(6) would impose a zero line flow and the line would be switched off. The direction of transmission flow is from bus m to bus n. Constraints (10)–(11) consider the preset limits for the standing phase angle difference. The standing phase angle difference must be within its limits

before an attempt is made to close breakers. Using (10)-(11), standing phase angle difference limits are imposed when a line is switched on. Using (12), the islanding is prevented. This constraint guarantees that at least one line is connected to each bus when all corresponding lines are switchable.

If the TS Feasibility Check is infeasible, an LP problem is formed by setting the state of switchable lines to 1 and considering transmission constraints (2)–(9). This LP problem minimizes the bus power mismatches for 24 h and forms Benders cuts for the UC master problem. The LP problem applies the UC schedule using (13):

$$P_{it} = \hat{P}_{it}\hat{I}_{it} \leftrightarrow \pi_{it} \qquad (i = 1, \dots, NG)$$
(13)

where  $\hat{P}_{it}$  and  $\hat{I}_{it}$  are fixed values calculated by the UC master problem. In (13), the arrow refers to the duality of the equality constraint, where  $\pi_{it}$  is the dual variable (also known as simplex multiplier) of equality constraint (13). Mathematically, this multiplier represents the marginal increment/decrement of the objective value when  $\hat{P}_{it}\hat{I}_{it}$  is changed. To form the Benders cut, this dual variable is needed. The solution of the LP problem will provide the hourly cuts for the UC master problem stated as

$$\hat{v}_t + \sum_{i=1}^{NG} \pi_{it} (P_{it} I_{it} - \hat{P}_{it} \hat{I}_{it}) \le 0.$$
(14)

In (14), the hourly bus power mismatch is noted as  $\hat{v}_t$ . The cut represents the coupled information on the unit generation dispatch and commitment state. The current violations can be mitigated by recalculating the hourly commitment states and the dispatch of generating units.

If the TS Feasibility Check solution is feasible, we will proceed to the Optimal TS Scheduling block as discussed next.

2) Optimal TS Scheduling: The Optimal TS Scheduling problem will calculate the optimal dispatch of generating units and the switchable line states in the base case and contingencies, given the hourly UC schedule

$$Min \sum_{i=1}^{NG} \sum_{t=1}^{NT} [F_{ci}(P_{it}^{c})\hat{I}_{it}]$$
(15)

which is subject to unit and system constraints. The unit constraints include generation bids (prices), and power generation and ramp up/down rate limits. System constraints include power balance and spinning/operating reserve requirements, as well as fuel and emission constraints. A complete list of constraints can be found in [13]. The objective function (15) considers fixed unit commitment states which are calculated in the UC master problem. In the case of contingencies, the objective is also subject to load balance (16) and generation limit (17) constraints:

$$\sum_{i=1}^{NG} P_{it}^{c} = D_{t} \qquad (t = 1, \dots, NT)$$
(16)

$$P_{i,\min}I_{it}UX_{it}^c \le P_{it}^c \le P_{i,\max}I_{it}UX_{it}^c$$
$$(t=1,\ldots,NT)(i=1,\ldots,NG)$$
(17)

where  $UX_{it}^c$  is the contingency state of unit *i* at time *t* in contingency *c*. Note that c = 0 represents base case. So, we would

have  $UX_{it}^0 = 1$ . In (16), the load balance equation is satisfied for each contingency. According to (17), when a unit is on outage (contingency), its generation would be zero.

The Optimal TS Scheduling is a MIP problem because of the binary states of switchable lines. At the first iteration, there are no constraints on line flows or switchable lines states. So, any values could be assigned to these variables. But in the subsequent iterations, the Benders cuts from the Transmission Contingencies Check establish constraints on switchable line states.

*3) Transmission Check in Contingencies:* The solution of the Optimal TS Scheduling block will be checked in the Transmission Contingencies Check as to whether a converged dc power flow solution can be obtained. So, we would have

$$Min \ w_t^c = \sum_{b=1}^{NB} (SL_{bt,1}^c + SL_{bt,2}^c).$$
(18)

This objective is subject to transmission security constraints (19)–(32):

$$P_{it}^c = \hat{P}_{it}^c \leftrightarrow \pi_{it}^c (i = 1, \dots, NG)$$
(19)

$$z_{lt}^c = \hat{z}_{lt}^c \leftrightarrow \mu_{lt}^c (l = 1, \dots, NL)$$
<sup>(20)</sup>

$$\sum_{i \in U_b} P_{it}^c - \sum_{l \in L_b} PL_{lt}^{sc} - \sum_{l \in L_b} PL_{lt}^{nsc} - Pd_{bt} + SL_{bt,1}^c - SL_{bt,2}^c = 0$$

$$(b = 1, \dots, NB)(s = 1, \dots, NS)(ns = 1, \dots, NNS)$$
(21)

$$PL_{lt}^{sc} - (\delta_{mt}^{c} - \delta_{nt}^{c}) / x_{l} + M(1 - z_{lt}^{c}) + M(1 - UY_{lt}^{c}) \ge 0$$

$$(l=1, \dots, NL)(s=1, \dots, NS)$$
(22)

$$PL_{lt}^{sc} - (\delta_{mt}^{c} - \delta_{nt}^{c})/x_{l} - M(1 - z_{lt}^{c}) - M(1 - UY_{lt}^{c}) \le 0$$
(23)  
(l=1,...,NL)(s=1,...,NS)

$$PL_{lt}^{sc} \le PL_{l,\max}z_{lt}^{c}UY_{lt}^{c}(l=1,\ldots,NL)(s=1,\ldots,NS)$$
 (24)

$$-PL_{lt}^{sc} \le PL_{l,\max} z_{lt}^c UY_{lt}^c (l=1,\ldots,NL) (s=1,\ldots,NS)$$
(25)

$$PL_{lt}^{nsc} - (\delta_{mt}^c - \delta_{nt}^c) / x_l + M(1 - UY_{lt}^c) \ge 0$$

$$(26)$$

$$(l=1,\ldots,NL)(n=1,\ldots,NNS)$$

$$PL_{lt}^{nsc} - (\delta_{mt}^c - \delta_{nt}^c) / x_l - M(1 - UY_{lt}^c) \le 0$$

$$(27)$$

$$(l=1,\ldots,NL)(ns=1,\ldots,NNS)$$

$$PL_{lt}^{nsc} \leq PL_{l,\max}UY_{lt}^{c}, (l=1,\ldots,NL)(ns=1,\ldots,NNS)$$
(28)

$$-PL_{lt}^{nsc} \le PL_{l,\max}UY_{lt}^c(l=1,\ldots,NL)(ns=1,\ldots,NNS)$$
(29)

$$\delta_{mt}^{c} - \delta_{nt}^{c} \le \Delta_{l} + M z_{l(t-1)}^{c} + M (1 - z_{lt}^{c}) (l = 1, \dots, NL)$$
(30)

$$\delta_{mt}^{c} - \delta_{nt}^{c} \ge -\Delta_{l} - M z_{l(t+1)}^{c} - M(1 - z_{lt}^{c})(l = 1, \dots, NL)$$
(31)

$$\sum_{l \in L_b} z_{lt}^c \ge 1(t=1,\dots,NT) \tag{32}$$

where  $\hat{P}_{it}^c$  and  $\hat{z}_{lt}^c$  are fixed values calculated by Optimal TS Scheduling. The power flow of switchable lines is obtained by (22)–(25). When a line is switched off, the line flow will be set to zero and removed from power flow equations. The power flow of nonswitchable lines is obtained by (26)–(29). In (22)–(29) the contingency state of line *l* at time *t* is considered by  $UY_{It}^c$ . Note that c = 0 represents base case. Therefore, we would have  $UY_{lt}^0 = 1$ . Standing phase angle difference limits are considered by (30) and (31), and islanding is prevented using (32) for buses in which all corresponding lines are switchable. Here (30) and (31) use the given values of switchable states. So there is no time-based coupling in this problem.

If the total mismatch is zero, the proposed generation dispatch and switchable line schedule provide a feasible power flow solution which satisfies transmission security constraints. Otherwise, the Benders cut (33) will be added to the Optimal TS Scheduling block for mitigating the violations in the next iteration:

$$\hat{w}_{t}^{c} + \sum_{i=1}^{NG} \pi_{it}^{c} (P_{it}^{c} - \hat{P}_{it}^{c}) + \sum_{l=1}^{NL} \mu_{lt}^{c} (z_{lt}^{c} - \hat{z}_{lt}^{c}) \le 0.$$
(33)

This cut represents the coupled information on the unit generation dispatch and switchable line schedule. If the current UC solution cannot mitigate the contingency c violations when the maximum number of iterations has reached, this contingency will be labeled as uncontrollable and the Benders cut (34) will be returned to the UC master problem for calculating a preventive generation schedule

$$\hat{w}_{t}^{c} + \sum_{i=1}^{NG} \pi_{it}^{c} (P_{it} I_{it} - \hat{P}_{it}^{c} \hat{I}_{it}) \le 0.$$
(34)

The controllable contingencies are handled by corrective actions in the Transmission Contingencies Check without requiring any revisions to the existing UC solution. The proposed solution procedure is given as follows.

Step 1) Solve the UC master problem.

Step 2) Given the UC schedule, check the TS feasibility. If the TS is feasible, then proceed to Step 4.

Step 3) Minimize bus power mismatches if the TS Feasibility Check is infeasible. Form the Benders cuts and go to Step 1.

Step 4) Use the Optimal TS Scheduling to calculate the generation dispatch and the switchable line states in the base case and contingencies.

Step 5) Use the Transmission Check block to minimize bus power mismatches by utilizing the Optimal TS Scheduling results. If the mismatch is not zero, add the Benders cuts to the Optimal TS Scheduling for the next iteration. The controllable contingencies are handled by Transmission Check block. However, the Benders cut for uncontrollable contingencies are added to Step 1 for the next UC iteration. Stop the process if the total mismatch is zero.

#### **III. NUMERICAL SIMULATIONS**

Two case studies for the six-bus system and the IEEE 118-bus system are analyzed to illustrate the performance of the proposed method. The proposed method was implemented on a 2.4-GHz personal computer using CPLEX 11.0 [26].



Fig. 3. Six-bus system.

# A. Six-Bus System

The six-bus system is shown in Fig. 3. The objective is to calculate the least cost of dispatch with an hourly fixed load. The characteristics of generators, lines, and the hourly load distribution over the 24-h horizon are given in Tables I-III, respectively. The load is distributed as 20%, 40%, and 40% among buses 3, 4, and 5, respectively. In Table I, the maximum sustained rate (MSR) and the quick start capability (QSC) of units are used to satisfy the reserve requirements of the system. The MSR and QSC are used to limit the spinning and operating reserves of the unit, respectively. The 10-min spinning reserve of a unit is the unloaded synchronized generation that can ramp up in 10 min. The spinning reserve of a unit cannot exceed the difference between its maximum capacity and current generation dispatch. This reserve is limited by the 10-min MSR. Operating reserve is the unloaded synchronized/unsynchronized generating capacity that can ramp up in 10 min. When a unit is on, its operating reserve is the same as spinning reserve. When a unit is off, its operating reserve is equal to its QSC. The following TS cases are considered:

Case 1: Base case UC (without contingencies)

Case 2: Outage of line 2-4 is considered in Case 1

Case 3: Outage of unit 2 is considered in Case 1

Case 4: Standing phase angle difference limits are considered in Case 1

The range of standing phase angle differences that a system can withstand mostly depends on the voltage level and is usually determined by steady-state and dynamic simulations. However in Cases 1, 2, and 3, it is assumed that the maximum standing phase angle is large enough to satisfy the associated constraints. In Case 4, the impact of standing phase angle difference on UC solution is examined.

*Case 1:* The UC with a subsequent dc network security check is used to find the results shown in Table IV. The cheaper unit 1 is on at all hours while unit 2 is used at peak hours to satisfy the remaining load and minimize the operating cost. Unit 2 is committed at hour 11 due to a load increase. The transmission network encounters flow violations on lines 1–4 and 4–5. Therefore, expensive unit 3 is turned on to help mitigate violations. Line 4–5 is congested at peak hours 15–19, which leads to a lower dispatch of unit 1 and a higher operating cost. The total operating cost is \$125 465.

The TS application will result in a similar schedule, but unit 3 is not committed in the entire scheduling horizon. This UC schedule will satisfy the TS feasibility check. So, the

TABLE I CHARACTERISTICS OF GENERATING UNITS

Unit No.		1	2	3
Bus No.		1	2	6
	$c (\$/MW^2h)$	0.014	0.020	0.086
Cost Coefficients	b (\$/MWh)	19.96	23	29.14
	a (\$)	200	150	50
Minimum Capacity	7 (MW)	100	10	10
Maximum Capacit	y (MW)	220	200	50
Startup Cost (\$)		50	40	0
Shutdown Cost(\$)		100	200	0
Minimum Up Time	e (h)	4	3	1
Minimum Down T	ime (h)	4	2	1
Ramp Up Rate (M	W/h)	40	30	20
Ramp Down Rate	(MW/h)	50	35	20
MSR (MW/min)		2	1.5	0.5
QSC (MW)		15	10	10
Initial Hour		+4	+3	+1
Initial Generation (	(MW)	140	20	10

TABLE II CHARACTERISTICS OF TRANSMISSION LINES

Line No.	From Bus	To Bus	X (pu)	Flow limit (MW)
1	1	2	0.170	140
2	1	4	0.258	110
3	2	3	0.037	150
4	2	4	0.197	140
5	3	6	0.018	130
6	4	5	0.037	50
7	5	6	0.140	140

TABLE III HOURLY LOAD DEMAND

Hour	Load	Spinning	Operating
пош	(MW)	Reserve (MW)	Reserve (MW)
1	175.19	2.63	12.26
2	165.15	2.48	11.56
3	158.67	2.38	11.10
4	154.73	2.32	10.83
5	155.06	2.33	10.85
6	160.48	2.40	11.23
7	173.39	2.60	12.14
8	177.60	2.85	13.33
9	186.81	3.09	14.39
10	206.96	3.26	15.20
11	228.61	3.43	16.00
12	236.10	3.54	16.52
13	242.18	3.63	16.95
14	243.60	3.66	17.05
15	248.86	3.73	17.42
16	255.79	3.84	17.91
17	256.00	3.84	17.92
18	246.74	3.70	17.27
19	245.97	3.69	17.22
20	237.35	3.56	16.62
21	237.31	3.56	16.62
22	232.67	3.41	15.90
23	195.93	3.02	14.07
24	195.60	2.95	13.78

model would proceed to the Optimal TS Scheduling block. Imposing fewer cuts in this case would result in a cheaper UC solution. Using TS, the total operating cost is slightly dropped

TABLE IV UC Schedule of Six-Bus System in Case 1

Unit										]	Ho	ur	's (	0-	24	)									
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
3	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE V LINE SCHEDULE OF SIX-BUS SYSTEM IN CASE 1 USING TS

Line	Hours (0-24)
2-4	- 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
4-5	- 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0

to \$125 362. At hour 11 when lines 1–4 and 4–5 are congested, lines 2–4 and 4–5 are switched off instead of turning on unit 3. Here, there will be no transmission loop remaining in the system and the existing lines could increase the flows to their limits. Table V shows the state of switchable lines in which lines 2–4 and 4–5 are mostly switched off. At peak hours 16–17, the line flow constraints cannot be satisfied without placing back the lines 2–4 and 4–5. The standing phase angle differences for the two lines are 10.24 and 1.85 degrees, respectively.

*Case 2:* In this case, the outage of line 2–4 is considered. The UC in Case 1 cannot satisfy the dc network security check. The new UC is mostly the same as the schedule in Case 1, but unit 2 is committed at hours 10 and 24. This UC will mitigate the flow violations and satisfy the transmission security with a total operating cost of \$125 848. The UC solution given in Case 1 will satisfy the optimal TS scheduling block with line switching. The unit schedule is the same as that of Case 1. The TS schedule is different from that of Case 1, where line 4–5 is in service at peak hours 16–17 and off otherwise. The standing phase angle difference of line 4–5 at hour 16 is 0.3 degree. The line 2–4 contingency is handled with a corrective action at the Transmission Contingencies Check block. The congestion of line 1–2 is mitigated at peak hours 16–17 with a total operating cost of \$125 470 (i.e., 0.3% improvement)

*Case 3:* When unit 2 is on outage, unit 1 will not be able to satisfy the hourly load. Therefore, unit 3 is committed additionally. The preventive schedule without TS is shown in Table VI which is obtained with a more expensive daily operating cost of \$126 413. However, the proposed TS approach provides a preventive UC schedule. Comparing the new schedule with that of Table VI, we learn that the unit 2 is committed at hour 13 instead of hour 20, and unit 3 is not committed at hour 10. The flow violations at this hour are mitigated by switching off lines 2–4 and 4–5. The operating cost with TS is \$126 271. The state of switchable lines before outage of unit 2 is shown in Table VII. However, after the outage of unit 2, both switchable lines 2–4 and 4–5 will be switched off for the entire scheduling horizon.

In these three cases, all lines are considered as switchable. However, only lines 2–4 and 4–5 are switched in these cases. In Fig. 3, switching off any of lines 2–3, 3–6, 4–5, and 5–6 will remove the loop 2–3–4–5–6–2 with better results. Since line 4–5 has the lowest capacity, it is often subject to congestion which makes the line a good candidate for switching. This conclusion is consistent with the proposed results. By switching off line 4–5, the line flow in the loop, i.e., those of lines 2–3, 3–6,

TABLE VI UC Schedule of Six-Bus System in Case 3 Without TS

Unit										]	Hc	ur	s (	0-2	24	)									
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

TABLE VII LINE SCHEDULE OF SIX-BUS SYSTEM IN CASE 3 USING TS BEFORE OUTAGE OF UNIT 2

Line	Hours (0-24)
2-4	- 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1
4-5	- 0 0 0 0 0 0 0 0 0 1 0 1 1 0 1 1 1 1 1

and 5–6, can be raised without affecting the flow on other lines. Next, one of lines in the other loop, i.e., lines 1–2, 1–4, and 2–4, could be a good choice for switching. Since unit 1 is the most economical and the largest unit, it will remain in service along with the attached lines. So, line 2–4 would be the best TS choice for this loop. With switching off both lines 2–4 and 4–5, there are still enough lines connected to bus 4 to supply its load. By considering lines 2–4 and 4–5 as switchable, loops would be relaxed while meeting the load balance requirements.

*Case 4:* In this case, the standing phase angle difference limits of the switchable lines are considered. The standing phase angle difference limit for switchable lines is considered to be 4 degrees. Imposing this limit, when the absolute value of the difference of phase angles in two sides of the switched line is less than its standing phase angle difference limit, the line cannot be switched back to the system.

In Case 1, both switchable lines 2–4 and 4–5 are switched back to the system at hour 16. The standing phase angle difference for these two lines is 10.24 and 1.85 degrees, respectively. Since the standing phase angle difference of line 2–4 is more than its limit, this line cannot be switched back at hour 16. Imposing this constraint to the problem, the switchable line 2–4 is switched back to the system one hour earlier, i.e., hour 15. At this hour, the line can be switched back to the system, since the standing phase angle difference is less than its limit. With this change in the state of switchable lines, the generation of unit 1 is decreased by 2.5 MW and the generation of unit 2 is increased by 2.5 MW. This change in dispatch leads to an insignificant increase in the total operating cost.

#### B. IEEE 118-Bus System

A modified IEEE 118-bus system is used to study the SCUC with TS. The system has 118 buses, 54 units, and 186 branches. The data for this system are found in motor.ece.iit.edu/data/SCUC\_118test.xls. Lines 30, 78, 90, 115, 151, 159, and 164 are considered switchable. Three cases are considered. The first case is the base case UC, and the second one is SCUC when considering contingencies. At the third case, the results of the proposed model are compared with those of an integrated model.

*Case 1:* In this case, the SCUC approach without TS is used to find the generating unit schedules with a total operating cost of \$1 081 320.36. Using TS, seven lines are considered as switchable. The UC schedule found in the first iteration of the problem cannot satisfy the TS feasibility and Benders cuts are

 TABLE VIII

 UC Schedule of the IEEE 118-Bus System in Case 1 Using TS

Unit											]	Но	ur	s (	0-2	24)	)									
1-2	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4-5	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
9	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
10-11	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
14	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
15	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
16	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
17	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
18	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
19	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
20-21	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
23	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
24-25	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
27-29	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	(	)	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0
31	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
32	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
33	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34-35	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
36	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
37	(	)	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	(	)	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0
41-42	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	(	)	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44-45	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	(	)	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0
48	(	)	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0
49	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	(	)	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0
52	(	)	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0
53	(	)	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
54	(	)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0

 TABLE IX

 Line Schedule of the IEEE 118-Bus System in Case 1 Using TS

Line	Hours (0-24)
30	- 1 1 0 0 0 1 1 0 0 0 1 0 1 0 0 0 0 0 1 1 0 1 1
78	- 0 1 0 0 0 1 <u>1</u> 0 0 <u>0</u> 0 <u>1</u> 1 0 0 0 0 <u>0</u> 0 <u>1</u> 1 <u>0</u> <u>1</u> <u>1</u>
90	- 1 1 0 0 0 1 1 0 0 0 1 1 1 1 1 1 1 1 1
115	- 0 1 1 1 1 1 1 0 0 1 0 1 0 1 1 0 1 0 0 1 1 1 1
151	- 1 1 1 1 1 1 1 0 0 0 1 1 0 0 1 1 1 1 1
159	- 1 0 0 0 0 0 1 1 0 0 1 1 1 1 1 1 1 1 1
164	- 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

added to the UC problem. The feasible schedule is found after two iterations and shown in Table VIII. The TS schedule is shown in Table IX with a total operating cost of \$1078155.86, which signifies a 0.30% improvement in the total operating cost as compared with the UC solution. The total execution time is 94 s.

*Case 2:* In this case, three simultaneous contingencies are considered. The contingencies include outages of unit 13, line 75–77, and line 85–89. Comparing the new schedule with that in Case 1, the states of 20 units have changed at different hours with a total operating cost of \$1 081 898. To apply the proposed



Fig. 4. Total operating cost comparison of integrated and decomposed models.



Fig. 5. Execution time comparison of integrated and decomposed models.

TS approach, the same lines as in Case 1 are considered switchable. The first UC result is feasible for the outage of line 75–77. So, the contingency of line 75–77 is controllable. Since the UC solution cannot lead to a feasible solution for the outages of unit 13 and line 85–89, these contingencies are uncontrollable. The obtained switchable line schedule is completely different from that in Case 1. The differences are highlighted in Table IX in comparison with those in Case 1. The corresponding UC solution is shown in Table VIII, where the highlighted values show the difference with Case 1. The total operating cost is \$1 080 846, which shows a 0.09% improvement. This solution is obtained in 136 s.

*Case 3:* To show the effectiveness of the proposed model in handling a larger number of switchable lines, the TS cases with integrated and decomposed formulation are compared. The number of switchable lines is changed from 0 to 50 with step of 10. Zero means that no switchable line exists in the system, while 50 represents that more than one fourth of the transmission lines in the system are switchable. The default relative optimality gap of CPLEX, i.e., 0.01%, is used as the stopping criterion. The total operating cost and execution time for these two models for different numbers of switchable lines are obtained and shown in Figs. 4 and 5, respectively.

In Fig. 4, the decomposed model does not outperform the integrated model from optimality point of view. The integrated model always has a similar or better total operating cost. However, the difference between the total operating costs of two models is negligible. When there is no line switching in

the system, the two models result in the same solution. Since the proposed model is an extension of the decomposed SCUC model, it is expected that in case of no line switching, the decomposed model finds the same solution as the integrated model. The worst case occurs at 40 switchable lines, where the difference is 0.02%.

In Fig. 5, for the small number of switchable lines, both models find the solution in a reasonable time. For instance for ten switchable lines, the decomposed model finds the solution in 1 min and the integrated model finds the solution in 4 min. However, as we increase the number of switchable lines, the execution time of the integrated model increases significantly, while that of decomposed model remains reasonable. In the case of 50 switchable lines, the solution of the decomposed model is obtained in 9 min, while that of integrated model is achieved in more than 80 min. This considerable increase in execution time is due to the large number of binary variables that should be handled by the integrated problem. However in the decomposed model, these binary variables are divided between the UC master problem and the TS subproblem. This division would help the large problems to find solutions in a reasonable execution time.

# **IV. CONCLUSIONS**

In this paper, TS was integrated with UC for solving the multi-interval optimal generation unit scheduling with security constraints. The Benders decomposition was employed to separate the problem into a UC master problem and a TS sub-problem. The UC master problem minimizes the total operating cost and calculates the optimal hourly commitment and dispatch of generation units subject to power balance and reserve requirements. The TS subproblem adjusts the hourly unit schedules and optimizes the switching state of lines to mitigate transmission violations in the base case and controllable and uncontrollable contingencies. The features of the proposed approach are listed as follows.

- The TS model can be used for congestion management. The TS application will lead to adjustments in line flows and congestion levels.
- The number of switchable lines was limited here. This assumption is consistent with practical switching applications. If we increase the number of switchable lines, TS may find better SCUC solutions which could also converge at slower rates.
- The applications of Benders decomposition to SCUC and TS would make the proposed approach more applicable to large power systems.
- The proposed TS approach is an extension of the conventional SCUC. Hence, if we ignore the line switching in the proposed approach, the final SCUC solution will be the same as that of the conventional SCUC.
- The proposed approach allows for the utilization of corrective modes before resorting to more expensive preventive solutions.

The UC and the Optimal TS Scheduling blocks use the MIP format. MIP problems use a predefined gap to determine the optimality of solution. If this gap is set to zero, the problem will find the optimal solution with a longer computation time.

Otherwise, the problem will be terminated with a near-optimal solution in a reasonable time. Usually a near-optimal solution is considered in real-time applications in power systems. Additional points for improving the proposed TS approach are listed as follows.

- Numerical methods may be developed for identifying the optimal set of switchable lines. Such methods may include additional strategies for system operations. For example in the IEEE 118-bus system, line 164 would be a good prospect for switching since it is switched off periodically.
- Practical line switching strategies could be considered like how often and how long a line could be on/off.
- Switching states of lines are employed in the Optimal TS Scheduling by means of cuts. These binary variables will be adjusted more accurately if we use additional constraints to make stronger connections between the switchable lines states and the generation dispatch. The approach could help the TS subproblem find the optimal solution in fewer iterations [27].

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