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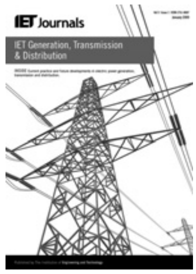

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Application of augmented Lagrangian relaxation to coordinated scheduling of interdependent hydrothermal power and natural gas systems

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Abstract: This study proposes an optimisation model for the coordinated scheduling of interdependent electric power and natural gas transmission systems from a joint operator's viewpoint. The objective is to minimise the coordinated social cost while satisfying network and temporal constraints of the two interdependent systems. The joint operator will coordinate hourly schedules to supply natural gas to loads or generate electric power. The authors consider the application of Lagrangian relaxation (LR) or augmented LR to relax the coupling constraints of the two systems. The Lagrangian dual is decomposed into the security-constrained unit commitment subproblem with the hydro coordination and the natural gas allocation subproblem. The application of LR for solving the coordinated problem could cause oscillations in the dual solution which is due to the non-convex characteristics of the coordinated problem represented by integer variables and network constraints. Moreover, with slight changes in multiplier values, the linear cost function of the natural gas well may result in a cycling behaviour of the gas well output between its max and min limits. To avoid numerical oscillations and improve the solution quality, the augmented LR with a piecewise linear approximation of quadratic penalty terms and the block descent coordination technique are proposed. The authors consider the 6-bus with 7-node and the 118-bus with 14-node systems to verify that the applicability of the proposed method to the coordinated scheduling of electric power and natural gas transmission systems.

Nomenclature

Index

i, j	index of power unit
gi	index of gas supplier including gas wells, storages and liquefied natural gas tanks
cm	index of compressor
el, gl	index of electricity load and residual gas load
t	index of hour
k	index of iteration
a, b	index of bus in power system
na, nb	index of node in gas network
l	index of branch in power system

Variables and functions

I	status indicator of generating unit
Y, Z	startup and shutdown indicator of unit
P	generation of unit
R	spinning reserve of unit
SU, SD	startup and shutdown cost of unit
X_{it}^{on}, X_{it}^{off}	up/down time of unit i at hour t

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GI	status indicator of gas well
GI^{o}, GI^{I}	indicator of releasing status and charging status of storage
GP	net output of gas well, storage
GP^{o}, GP^{I}	releasing and charging gas flow of storage
F_{ec},()	unit's production cost function
F_{gc},()	gas supplier's production cost function
F_{ef},()	natural gas consumption of gas-fired unit
F_{cf},()	natural gas consumption of compressor
F_{h},()	power-water discharge function
SV, HV	volume of gas storage and hydro reservoir
ELS, GLS	not served electricity and residual gas load
EL, GL	scheduled electricity and gas load
Pf_{l}	power flow through branch l
PN	pressure of node in gas system
GF_{nanb}	gas flow from node na to nb
CH	horsepower of compressor
$\boldsymbol{\gamma}$	control angel of phase shifter
θ	bus voltage angle
q	water discharge of hydro unit
s	spillage of hydro unit
η	step size to update Lagrangian multiplier
w	natural in flow to the reservoir of hydro unit
λ	Lagrangian multiplier
ω	penalty factor
$\boldsymbol{x}$, $\boldsymbol{y}$	vector of variables in power system and natural gas system
$\boldsymbol{x}_{c}$, $\boldsymbol{y}_{c}$	sub-vector of $\boldsymbol{x}$, $\boldsymbol{y}$ representing variables in coupling constraints
$\boldsymbol{e}(\boldsymbol{x}_{c})$, $\boldsymbol{g}(\boldsymbol{y}_{c})$	functions in coupling constraints

Constants

ρ	price of well-head of natural gas well, charging gas storage or releasing gas storage
σ	price of not served electricity and gas load
α, β	parameters of augmented LR
NT	scheduling period
$\boldsymbol{RC}$	connection matrix for cascaded hydro reservoirs
x_{ab}	reactance between buses a and b
$PF_{l,\max}$	power flow limits of branch l
EL_{D}, GL_{D}	estimated electricity and residual gas load
R_{D}	required spinning reserve of system
P_{Loss}	power transmission loss
GU	set of gas-fired units
GS	set of gas storages
UR, DR	max ramp up/down rate
T^{on}, T^{off}	min on/off time of unit
$P_{\min}$, $P_{\max}$	min/max capacity of a unit
$GP_{\min}$, $GP_{\max}$	max/min net output of gas well and storage
$PN_{\min}$, $PN_{\max}$	max/min pressure
$SV_{\max}$, $SV_{\min}$	upper/lower volume limit of hydro reservoir
$HV_{\max}$, $HV_{\min}$	upper/lower volume limits of storage
$PR_{\min}$, $PR_{\max}$	max/min pressure ratio of compressor
$CH_{\min}$, $CH_{\max}$	max/min horsepower of compressor
$\boldsymbol{A}$	bus-generator incidence matrix
$\boldsymbol{B}$	bus-electrical load incidence matrix
$\boldsymbol{C}$	bus-branch incidence matrix
$\boldsymbol{GA}$	node-gas supplier incidence matrix
$\boldsymbol{GB}$	node-gas load incidence matrix
$\boldsymbol{GK}$	node-gas branch incidence matrix
$\boldsymbol{GD}$	gas withdrawing node-compressor incidence matrix
$\boldsymbol{GE}$	gas load-power unit index incidence matrix

1 Introduction

In the last few decades, the number of gas-fired generating unit installations in the world has grown dramatically, which is mostly based on three reasons [1]. First, new combined-cycle gas units demonstrate higher economics over other fossil generating units. Second, gas-fired units have lower environmental impacts. Third, the rapid growth and the installation of volatile and intermittent renewable generating units in electric power systems would require additional generation reserves provided by fast response gas-fired units. According to statistics, the installed natural gas-fired generating capacity in ERCOT, Florida and ISO New England represented 60, 51 and 38% of the total generation capacity, respectively [1, 2]. In Europe and South America, natural gas-fired generation would also account for a considerable proportion of the total generation capacity. The electricity and gas infrastructures are highly interdependent. Accordingly, the security and economics of electric power systems are influenced by the economic allocation of natural gas resources as well as the secure operation of natural gas transmission systems. The natural gas transmission systems are also influenced by the hourly scheduling and the optimal operation of electric power systems.

In [3] the interdependency of gas and electricity was addressed. In [4] a generalised network flow model of the

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US integrated energy system was proposed considering natural gas, coal and power infrastructures. In [5] a non-linear continuous optimisation model was proposed for a coordinated flow in electricity and natural gas infrastructures. The short-term operation planning of integrated natural gas and hydrothermal power systems with linear gas transmission constraints was considered in [6], where the unit commitment (UC) problem was solved by Lagrangian relaxation (LR) and dynamic programming methods. The impact of the natural gas transmission system on electric power markets was discussed in [7]. Our previous paper [8] presented a comprehensive scheduling model by incorporating non-linear natural gas transmission constraints and contracts. The natural gas usage limits of gas-fired generating units were implicitly determined by the feasible adjustment range of the natural gas network and priority orders of gas load contracts. The Benders decomposition was used to apply the hourly UC results to separate blocks of electric power and natural gas transmission constraints. However, our previous model considered the viewpoints of the ISO and vertically integrated utility operators. Furthermore, operating costs of compressors and natural gas wells, and residual gas load models were not directly considered in the objective function.

In this paper, we propose a coordinated scheduling model from a joint operator's viewpoint as shown in Fig. 1. The coordination model is a mixed-integer non-linear optimisation problem in which the objective function will minimise the social cost of electric power and natural gas systems. In our proposed model, the joint operator is an independent organisation, which could operate outside the traditional jurisdictions of gas and electric power operators and would pursue the overall interest of coordinated energy systems. Natural gas resources will be allocated optimally to either supply gas loads or gas-fired generating units. The two systems have a decomposable structure and we consider the LR method as the decomposition strategy of the coordination problem. The coupling constraints between the electric power system and the natural gas transmission system are relaxed by Lagrangian multipliers and dualised into the objective function. The LR method is divided into two phases. The first phase is to solve the dual problem. However, the solution of phase one may not be feasible when considering the primal problem. Thus, the phase two of the dual problem will seek a feasible solution based on the solution of phase one as shown in Fig. 1. The relaxed primal problems are decomposed into security-constrained unit commitment subproblem with the hydro coordination (SCUC) and gas allocation subproblems that can be solved independently but in coordination. The methodologies for SCUC and natural gas allocation problems were developed in [9, 10], which incorporate the LR framework in our proposed model to solve the mixed-integer non-linear subproblems individually.

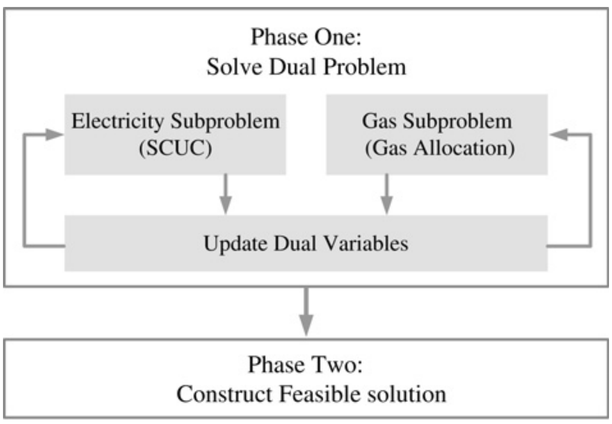

Figure 1 *Augmented LR-based electricity-gas scheduling coordination*

We demonstrate that the LR approach in our coordination model will not exhibit a satisfactory convergence. The non-convex characteristics of the coordinated problem will result in the oscillation of dual solution, which is due to integer variables and network constraints. Moreover, with slight changes in the multipliers, the linear cost function of the natural gas well may lead to a cycling behaviour of gas well output between its max and min values. Accordingly, the violation of relaxed constraints cannot be alleviated iteratively. Hence, the augmented LR method with piecewise linear approximation of quadratic penalty term is used for preventing numerical oscillations and improving the quality of dual solution. The Lagrangian dual will no longer be decomposable after introducing inseparable penalty terms. So we use the block descent coordination (BDC) technique to deal with this problem and solve the decomposed SCUC and gas allocation subproblems sequentially.

The rest of the paper is organised as follows. Section 2 proposes formulations of integrated scheduling model. Section 3 presents the LR and augmented LR-based methodology to implement coordination procedures. Numerical cases are studied in Section 4. The conclusion is drawn in Section 5.

2 Scheduling coordination model

2.1 Modelling outline

Our proposed model focuses on the steady-state analyses of the electricity and the natural gas systems. Both include integer variables with non-linear constraints. The outline of our proposed model is described as the following optimisation problem

Max Social welfare or Min Social cost

s.t.

(a) Power balance and reserve requirements.

(b) Individual generator constraints (including min on/off time, min/max generation capacity, startup/ shutdown characteristics, ramp rate limits, etc.).

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(c) Power transmission constraints.

(d) Gas source limits and gas storage constraints.

(e) Natural gas network constraints.

(f) Electricity – gas coupling constraints.

The objective function is to minimise the social cost, which is the sum of electricity and natural gas operating costs over the scheduling period as shown in (1)

$$\mathrm{Min}(\mathrm{EC}+\mathrm{GC}) \tag{1}$$

In our proposed model, natural gas-fired generating units will consider maintenance and crew costs and fuel costs will be managed by the joint operator. Therefore EC in (2) represents all operating costs of non-gas-fired units, electricity load-not-served penalty as well as non-fuel operating costs of gas-fired units.

$$\begin{aligned}\mathrm{EC} = &\sum_t \left[\sum_{i\notin \mathrm{GU}} F_{\mathrm{ec},i}(P_{it}, I_{it}) + \mathrm{SU}_{it} + \mathrm{SD}_{it}\right] \\ &+ \sum_t \sum_{\mathrm{el}} \sigma_{\mathrm{el}} \mathrm{ELS}_{\mathrm{el}t} \\ &+ \sum_t \left[\sum_{i\in \mathrm{GU}} F_{\mathrm{ec},i}(P_{it}, I_{it}) + \mathrm{SU}_{it} + \mathrm{SD}_{it}\right]\end{aligned} \tag{2}$$

The fuel cost of natural gas-fired units, which depends on the individual unit consumptions, will be implicitly considered in the natural gas allocation cost (GC). GC is represented in (3), which includes operating costs of gas well, liquefied natural gas (LNG) and gas storage as well as penalty costs for the residual natural gas load-not-served. σ_{gl} is the penalty price corresponding to residual gas loads, indicating their incremental costs and priority orders.

$$\mathrm{GC} = \sum_t \sum_{\mathrm{gi}} F_{\mathrm{gc,gi}}(\cdot) + \sum_t \sum_{\mathrm{gl}\notin \mathrm{GU}} \sigma_{\mathrm{gl}} \mathrm{GLS}_{\mathrm{gl}t} \tag{3}$$

The joint operator will coordinate the operation schedule to pursue the overall interests of coupled electricity and natural gas systems. The optimal allocation of natural gas to residual loads or gas-fired generating units is determined by market demands and relative incremental costs. For instance, joint operators will supply more natural gas to power plants, if the proposed supply of fuel to gas-fired generating units will result in the additional commitment of expensive generators. Also, higher penalty costs for not supplying the residual gas loads will lead to a larger supply of natural gas to such loads. Here, we may consider gas-fired units to provide a generation service to the coordinated electricity and natural gas systems while being compensated for their maintenance or crew costs. The joint operator will then perform a coordination of fuel consumption by electric and natural gas systems.

2.2 Power system constraints

1. Power balance and reserve constraint

$$\sum_i P_{it} I_{it} + \sum_{\mathrm{el}} \mathrm{ELS}_{\mathrm{el}t} = \sum_{\mathrm{el}} \mathrm{EL}_{D,\mathrm{el}t} + P_{\mathrm{Loss},t} \tag{4}$$

$$\sum_i R_{it} I_{it} \ge R_D \tag{5}$$

2. Individual unit constraints

Min on/off time

$$[X^{\mathrm{on}}_{i(t-1)} - T^{\mathrm{on}}_i]\,[I_{i(t-1)} - I_{it}] \ge 0 \tag{6}$$

$$[X^{\mathrm{off}}_{i(t-1)} - T^{\mathrm{off}}_i]\,[I_{it} - I_{i(t-1)}] \ge 0 \tag{7}$$

Ramping rate limits

$$P_{it} - P_{i(t-1)} \le Y_{it}\, P_{\min,i} + (1 - Y_{it})\,\mathrm{UR}_i \tag{8}$$

$$P_{i(t-1)} - P_{it} \le Z_{it}\, P_{\min,i} + (1 - Z_{it})\,\mathrm{DR}_i \tag{9}$$

Max/Min power generation

$$P_{\min,i}\, I_{it} \le P_{it} \le P_{\max,i}\, I_{it} - R_{it} \tag{10}$$

A more detailed formulation of such constraints including emission and fuel constraints is given in [9]. Either a mode or a component model [1] may be used for the simulation of combined-cycle generating units.

3. Hydro unit and reservoir constraints

Power–water discharge conversion relationship

$$P_{it} = F_{h,i}(q_{it}, I_{it}) \tag{11}$$

Water discharge limits

$$q_{i,\min}\, I_{it} \le q_{it} \le q_{i,\max}\, I_{it} \tag{12}$$

Reservoir volume limits

$$\mathrm{HV}_{i,\min} \le \mathrm{HV}_{it} \le \mathrm{HV}_{i,\max} \tag{13}$$

$$\mathrm{HV}_{i,t=0} = \mathrm{HV}_{0,i}, \quad \mathrm{HV}_{i,t=\mathrm{NT}} = \mathrm{HV}_{\mathrm{NT},i} \tag{14}$$

Water balance constraint for cascaded hydro units

$$\mathrm{HV}_{it} = \mathrm{HV}_{i,t-1} - q_{it} - s_{it} + w_{it} + \boldsymbol{RC}_{ij}\, q_{j(t-\tau_j)} \tag{15}$$

where $q_{j(t-\tau)}$ represents the delayed water discharge to hydro unit i from other hydro units j.

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4. Power transmission constraints

$$\begin{aligned} &\boldsymbol{C} \cdot \mathrm{PF} = \boldsymbol{A} \cdot \boldsymbol{P} - \boldsymbol{B} \cdot (\mathrm{EL} - \mathrm{ELS}) \\ &\mathrm{PF}_l = \frac{\theta_a - \theta_b - \gamma_{ab}}{x_{ab}} \quad (a, b \in l) \\ &|\mathrm{PF}_l| \le \mathrm{PF}_{l,\max} \\ &\boldsymbol{\gamma}_{\min} \le \boldsymbol{\gamma} \le \boldsymbol{\gamma}_{\max} \\ &\theta_{\mathrm{ref}} = 0 \end{aligned} \tag{16}$$

2.3 Natural gas constraints

1. Gas well and storage constraints: The natural gas source is represented by gas well, gas storage and LNG tank, which demonstrate distinct prices and operating characteristics.

Cost of gas well is given as

$$F_{\mathrm{gc,gi}}(\cdot) = \rho_{\mathrm{gi}t}\,\mathrm{GP}_{\mathrm{gi}t}, \quad \forall \mathrm{gi} \notin \mathrm{GS} \tag{17}$$

Gas well and LNG source satisfy the following constraint

$$\mathrm{GI}_{\mathrm{gi}t}\,\mathrm{GP}_{\mathrm{gi,\,min}} \le \mathrm{GP}_{\mathrm{gi}t} \le \mathrm{GI}_{\mathrm{gi}t}\,\mathrm{GP}_{\mathrm{gi,\,max}}, \quad \forall \mathrm{gi} \notin \mathrm{GS} \tag{18}$$

Natural gas storage or LNG tank represents supplemental gas sources. The gas storage operation may switch among three exclusive modes, that is, releasing gas, charging gas and being off. When charging or releasing gas, additional operating costs will be considered as shown in (19)

$$F_{\mathrm{gc,gi}}(\cdot) = \rho^{\mathrm{I}}_{\mathrm{gi}t}\,\mathrm{GP}^{\mathrm{I}}_{\mathrm{gi}t} + \rho^{\mathrm{O}}_{\mathrm{gi}t}\,\mathrm{GP}^{\mathrm{O}}_{\mathrm{gi}t}, \quad \forall \mathrm{gi} \in \mathrm{GS} \tag{19}$$

Max/Min flow rate while releasing or charging gas

$$\mathrm{GI}^{\mathrm{I}}_{\mathrm{gi}t}\,\mathrm{GP}^{\mathrm{I}}_{\mathrm{gi,\,min}} \le \mathrm{GP}^{\mathrm{I}}_{\mathrm{gi}t} \le \mathrm{GI}^{\mathrm{I}}_{\mathrm{gi}t}\,\mathrm{GP}^{\mathrm{I}}_{\mathrm{gi,\,max}}, \quad \forall \mathrm{gi} \in \mathrm{GS} \tag{20}$$

$$\mathrm{GI}^{\mathrm{O}}_{\mathrm{gi}t} \cdot \mathrm{GP}^{\mathrm{O}}_{\mathrm{gi,\,min}} \le \mathrm{GP}^{\mathrm{O}}_{\mathrm{gi}t} \le \mathrm{GI}^{\mathrm{O}}_{\mathrm{gi}t} \cdot \mathrm{GP}^{\mathrm{O}}_{\mathrm{gi,\,max}}, \quad \forall \mathrm{gi} \in \mathrm{GS} \tag{21}$$

Net output of gas storage is the difference between releasing and charging gas flow as shown in (22)

$$\mathrm{GP}^{\mathrm{O}}_{\mathrm{gi}t} - \mathrm{GP}^{\mathrm{I}}_{\mathrm{gi}t} = \mathrm{GP}_{\mathrm{gi}t}, \quad \forall \mathrm{gi} \in \mathrm{GS} \tag{22}$$

In addition, there is a volume balance constraint for each storage (e.g. hydro reservoir)

$$\mathrm{SV}_{\mathrm{gi}t} - \mathrm{SV}_{\mathrm{gi}(t+1)} = \mathrm{GP}_{\mathrm{gi}t}, \quad \forall \mathrm{gi} \in \mathrm{GS} \tag{23}$$

Volume of gas storage is restricted as

$$\mathrm{SV}_{\mathrm{gi,\,min}} \le \mathrm{SV}_{\mathrm{gi}t} \le \mathrm{SV}_{\mathrm{gi,\,max}}, \quad \forall \mathrm{gi} \in \mathrm{GS} \tag{24}$$

$$\mathrm{SV}_{i,t=0} = \mathrm{SV}_{0,i} \quad \mathrm{SV}_{i,t=\mathrm{NT}} = \mathrm{SV}_{\mathrm{NT},i} \tag{25}$$

2. Gas transmission constraints: A steady-state gas transmission model is built based on the nodal gas mass balance, which indicates that the gas flow injected to a node is equal to the gas flow out of the node as shown in (27). The natural gas pressure associated with each node is satisfied within a reasonable range (28).

$$\begin{aligned} &\sum_{\mathrm{gi}} \mathrm{GA}_{na,\mathrm{gi}}\,\mathrm{GP}_{\mathrm{gi}} - \sum_{\mathrm{gl}} \mathrm{GB}_{na,\mathrm{gl}}(\mathrm{GL}_{\mathrm{gl}} - \mathrm{GLS}_{\mathrm{gl}}) \\ &\quad - \sum_{nb} \mathrm{GK}_{na,nb}\,\mathrm{GF}_{nanb} + \sum_{\mathrm{cm}} \mathrm{GD}_{na,\mathrm{cm}}\,F_{nf,\mathrm{cm}}(\cdot) = 0 \end{aligned} \tag{26}$$

The Weymouth equation [11] indicates the flow in a pipeline extending from gas node na to gas node nb is modelled as

$$\mathrm{GF}_{nanb} = \mathrm{sgn}(\mathrm{PN}_{na}, \mathrm{PN}_{nb})\,C_{mn}\sqrt{|\mathrm{PN}^2_{na} - \mathrm{PN}^2_{nb}|} \tag{27}$$

$$\mathrm{PN}_{\min,na} \le \mathrm{PN}_{na} \le \mathrm{PN}_{\max,na} \tag{28}$$

where C_{mn} is the pipeline constant that depends on temperature, length, diameter, friction and gas composition.

For driving the natural gas flow from providers to gas loads, compressors are built at intervals along the gas pipeline to compensate the pressure loss [5, 11]. The gas flow through centrifugal compressor is governed by (30)–(32)

$$\mathrm{GF}_{nanb} = \mathrm{sgn}(\mathrm{PN}_{na}, \mathrm{PN}_{nb}) \cdot \frac{\mathrm{CH}_{\mathrm{cm}}}{k1_{\mathrm{cm}} - k2_{\mathrm{cm}}\mathrm{PR}^{k3_{\mathrm{cm}}}} \tag{29}$$

$$\mathrm{CH}_{\mathrm{min,cm}} \le \mathrm{CH}_{\mathrm{cm}t} \le \mathrm{CH}_{\mathrm{max,cm}} \tag{30}$$

$$\mathrm{PR}_{\min} \le \frac{\max(\mathrm{PN}_{na}, \mathrm{PN}_{nb})}{\min(\mathrm{PN}_{na}, \mathrm{PN}_{nb})} \le \mathrm{PR}_{\max} \tag{31}$$

where $k1_{\mathrm{cm}}$, $k2_{\mathrm{cm}}$ and $k3_{\mathrm{cm}}$ are empirical parameters corresponding to the compressor design.

2.4 Electricity–natural gas coupling constraints

A gas-fired power plant represents the linkage between natural gas and electricity systems. The gas consumption of a generation unit is a function of its hourly power generation stated as

$$\mathrm{GL}_{\mathrm{gl}t} = \sum_i \boldsymbol{GE}_{\mathrm{gl}i}\,F_{\mathrm{ef},i}(P_{it}, I_{it}), \quad \forall i \in \mathrm{GU} \tag{32}$$

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3 Solution of coordinated scheduling model

3.1 Coordinated scheduling by LR

We try the LR method first. A group of equations in (32) represents the electricity–gas coordination problem. The two systems have a decomposable structure and we consider the LR method as the decomposition strategy of the coordination problem (1)–(32). The LR method is divided into two phases as shown in Fig. 1. For the sake of clarity, in Section 3 we use vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ to represent electricity and natural gas system variables, respectively. The coupling constraints are expressed as in (33) instead of (32), in which $\boldsymbol{x}_c$, $\boldsymbol{y}_c$ are subvectors of $\boldsymbol{x}$ and $\boldsymbol{y}$ for representing variables in coupling constraints.

$$\boldsymbol{e}(\boldsymbol{x}_c) - \boldsymbol{g}(\boldsymbol{y}_c) = 0 \tag{33}$$

In the Lagrangian function (34), the coupling constraints (33) are relaxed and incorporated into the objective function using the following Lagrangian multipliers

$$\mathrm{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda}) = \mathrm{EC}(\boldsymbol{x}) + \mathrm{GC}(\boldsymbol{y}) + \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{e}(\boldsymbol{x}_c) - \boldsymbol{\lambda}^{\mathrm{T}}\boldsymbol{g}(\boldsymbol{y}_c) \tag{34}$$

The relaxed primal problem (35) is formulated in terms of minimising the Lagrangian function subject to constraints (4)–(32). In (35) $\phi(\boldsymbol{\lambda})$ is defined as the Lagrangian dual function with respect to $\boldsymbol{\lambda}$.

$$\phi(\boldsymbol{\lambda}) = \underset{x,y}{\mathrm{Min}}\{\mathrm{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda})|(4)\text{–}(31)\} \tag{35}$$

The resulting max–min problem is the following dual problem

$$\underset{\boldsymbol{\lambda}}{\mathrm{Max}}\,\underset{x,y}{\mathrm{Min}}\{\mathrm{L}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\lambda})|(4)\text{–}(31)\} \tag{36}$$

The difference between the optimal objective function values of primal and dual problems (36) is the duality gap.

For a given $\boldsymbol{\lambda}^{(k)}$, the Lagrangian dual (35) of the primal problem is decomposed into the independent SCUC and the gas allocation subproblems as shown in (37) and (38).

$$\underset{x}{\mathrm{Min}}\{\mathrm{EC}(\boldsymbol{x}) + \boldsymbol{\lambda}^{(k)}\,\boldsymbol{e}(\boldsymbol{x}_c)|(4)\text{–}(16)\} \tag{37}$$

$$\underset{y}{\mathrm{Min}}\{\mathrm{GC}(\boldsymbol{y}) - \boldsymbol{\lambda}^{(k)}\,\boldsymbol{g}(\boldsymbol{y}_c)|(17)\text{–}(31)\} \tag{38}$$

Since $\boldsymbol{\lambda}^{(k)}$ may not be the optimal solution of the dual problem (36), the dual cost $\phi(\boldsymbol{\lambda}^{(k)})$ resulted from the solution of subproblems (37) and (38) will produce a lower bound for the optimal solution of the dual problem (36). According to the weak duality theory, (39) is satisfied where $\boldsymbol{x}^*, \boldsymbol{y}^*$ is the optimal solution of primal problem and λ^* is the optimal solution of the dual problem.

$$\phi(\boldsymbol{\lambda}^{(k)}) \le \phi(\boldsymbol{\lambda}^*) \le \mathrm{EC}(\boldsymbol{x}^*) + \mathrm{GC}(\boldsymbol{y}^*) \tag{39}$$

The phase one procedure of LR is to update Lagrangian multipliers $\boldsymbol{\lambda}$ and then solve the resulting small-scale optimisation problem (37), (38) iteratively. The dual cost would increase gradually until changes of $\boldsymbol{\lambda}$ or $\boldsymbol{x}_c$ $\boldsymbol{y}_c$ are relatively small. The Lagrangian multipliers would be updated in the direction of the dual cost increment. The subgradient method shown in (40) is the most popular one. The parameter $\eta^{(k)}$ in (40) represents the step size that would satisfy (41) for convergence [12, 13]. Since $\phi(\boldsymbol{\lambda}^*)$ is generally unknown before the dual problem is solved, we use the estimated value of $\phi(\boldsymbol{\lambda}^*) - \phi(\boldsymbol{\lambda}^{(k)})$.

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \eta^{(k)}[\boldsymbol{e}(\boldsymbol{x}_c) - \boldsymbol{g}(\boldsymbol{y}_c)] \tag{40}$$

$$0 < \eta^{(k)} < \frac{\phi(\boldsymbol{\lambda}^*) - \phi(\boldsymbol{\lambda}^{(k)})}{\|\boldsymbol{e}(\boldsymbol{x}_c) - \boldsymbol{g}(\boldsymbol{y}_c)\|^2} \tag{41}$$

where $\|\cdot\|$ represents Euclidian norm.

3.2 Coordinated scheduling by augmented LR

The proposed LR method has demonstrated a few drawbacks as follows. The linear production cost function of gas well and piecewise linear generator production cost function will make the electricity and gas subproblems oscillate between maximum and minimum outputs. Moreover, non-convex characteristics of our coordination problem with integer variables and non-linear network constraints will create a large duality gap, which will make it difficult to find a good dual solution. Based on our experience, a good dual solution with a lower degree of violation will result in an optimal primal solution. Furthermore, the LR application in our case will cause oscillations in the solution of dual problem, which is due to the linearity of the price function of gas wells, storage and contracts. A similar phenomenon is recognised in the solution of hydrothermal coordination problem [14, 15]. In the following, the augmented LR is used that introduces penalty terms to smooth out the dual function and alleviate numerical oscillations. We relax the coupling constraint (33) in an augmented Lagrangian fashion in (42), where ω is a positive penalty factor.

$$\begin{aligned}\mathrm{A}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\omega}, \boldsymbol{\lambda}) = {} & \mathrm{EC}(\boldsymbol{x}) + \mathrm{GC}(\boldsymbol{y}) + \boldsymbol{\lambda}^{\mathrm{T}}[\boldsymbol{e}(\boldsymbol{x}_c) - \boldsymbol{g}(\boldsymbol{y}_c)] \\ & + \omega\|\boldsymbol{e}(\boldsymbol{x}_c) - \boldsymbol{g}(\boldsymbol{y}_c)\|^2\end{aligned} \tag{42}$$

The augmented Lagrangian function (42) cannot be decomposed as it contains an inseparable cross penalty term, whose variables belong to both power and gas constraints. To make this term separable, [16] uses the auxiliary problem principle to linearise the penalty term.

An alternative is to use the BCD method, which is a non-linear Gauss–Seidel-type method [13]. The BCD would solve the subproblems (43) and (44) sequentially. Here, when minimising one of the subproblems, the coupling variables of the other one appears in an inseparable penalty term, which will be fixed based on the latest solution $\tilde{\boldsymbol{x}}_c, \tilde{\boldsymbol{y}}_c$ of subproblems. In this paper, BCD method is adopted.

$$\operatorname*{Min}_{\boldsymbol{x}}\left\{\mathrm{EC}(\boldsymbol{x})+\boldsymbol{\lambda}^{(k)\mathrm{T}}\,\boldsymbol{e}(\boldsymbol{x}_c)+\frac{1}{2}\omega^{(k)}\|\boldsymbol{e}(\boldsymbol{x}_c)-\boldsymbol{g}(\tilde{\boldsymbol{y}}_c)\|^2\right\} \quad \text{such that (4)–(16)} \tag{43}$$

$$\operatorname*{Min}_{\boldsymbol{y}}\left\{\mathrm{GC}(\boldsymbol{y})-\boldsymbol{\lambda}^{(k)\mathrm{T}}\,\boldsymbol{g}(\boldsymbol{y}_c)+\frac{1}{2}\omega^{(k)}\|\boldsymbol{e}(\tilde{\boldsymbol{x}}_c)-\boldsymbol{g}(\boldsymbol{y}_c)\|^2\right\} \quad \text{such that (17)–(31)} \tag{44}$$

This procedure may create high-order terms for subproblems (43) and (44). In this paper, a piecewise linear approximation with respect to quadratic penalty terms is used. The dual problem is formulated as

$$\phi_\omega(\boldsymbol{\lambda}^*)=\operatorname*{Max}_{\boldsymbol{\lambda}}\{\operatorname*{Min}_{\boldsymbol{x},\boldsymbol{y}}\mathrm{A}(\boldsymbol{x},\boldsymbol{y},\omega,\boldsymbol{\lambda})|(4)\text{–}(31)\} \tag{44}$$

The updating of Lagrangian multipliers can still use (40). The iterative solution steps for the augmented LR-based coordination algorithm are discussed as follows:

1. Initiate the Lagrangian multipliers $\boldsymbol{\lambda}^{(0)}$ and penalty factors $\omega^{(0)}$, $k=0$.

2. For the given $\boldsymbol{\lambda}^{(k)}\omega^{(k)}\tilde{\boldsymbol{y}}_c$, solve the electricity subproblem (43). Update $\tilde{\boldsymbol{x}}_c=\boldsymbol{x}_c^{(k)}$.

3. Solve the gas subproblem (45) based on $\boldsymbol{\lambda}^{(k)}$, $\omega^{(k)}$ and $\tilde{\boldsymbol{x}}_c$. Update $\tilde{\boldsymbol{y}}_c=\boldsymbol{y}_c^{(k)}$.

4. Update the Lagrangian multipliers $\boldsymbol{\lambda}$ based on the subgradient method (40).

5. If $\|\boldsymbol{e}(\boldsymbol{x}_c^{(k)})-\boldsymbol{g}(\boldsymbol{y}_c^{(k)})\|>\alpha\|\boldsymbol{e}(\boldsymbol{x}_c^{(k-1)})-\boldsymbol{g}(\boldsymbol{y}_c^{(k-1)})\|$ Update $\omega^{(k+1)}=\beta\omega^{(k)}$, $\beta>1$.

6. If $\|\boldsymbol{x}_c^{(k)}-\boldsymbol{y}_c^{(k)}\|\le\varepsilon$, the final primal-dual solution is calculated as $(\boldsymbol{x}^{(k)},\boldsymbol{y}^{(k)},\boldsymbol{\lambda}^{(k)})$. Otherwise, $k=k+1$.

7. If the iteration number k is larger than the pre-specified number, go to step 8. Otherwise, go to step 2.

8. Construct the final feasible solution to the primal problem based on the obtained best solution to the dual problem.

3.3 Solution of SCUC and gas allocation subproblems

The common points of SCUC [9] and gas allocation [10] optimisation subproblems are represented by their types and structures. First, both are mixed-integer programming subproblems. Second, both have transmission network constraints and hold an L-shaped structure. For large-scale applications, the network security check is usually separated from the economic resource dispatch by either the Benders decomposition or the sensitivity analysis (i.e. power transfer distribution factor). The framework for the solution of SCUC or gas allocation subproblems is given in Fig. 2. More detailed formulations are provided in [8, 9]. The methodologies that may be engaged for the solution of SCUC or natural gas allocation subproblem do not change the overall framework of the proposed LR-based coordination strategy. Such methodologies can also be incorporated into our algorithm to solve the two subproblems [17, 18].

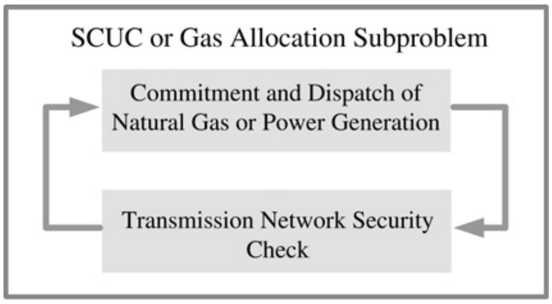

Figure 2 *Framework for the solution of SCUC/gas allocation subproblem*

3.4 Calculation of feasible solution

In some cases, the convergence of dual problem is quick and fairly reliable, whereas in other cases the solution tends to exhibit a cycling behaviour, especially when using the LR approach. Usually, the iterative process is terminated after a pre-specified number of iterations. However, even the dual solution resulted from the last iteration may still be infeasible in the primal case, which is due to smaller violations of coupling constraints. Accordingly, we need to construct a feasible solution in the phase two of dual solution shown in Fig. 1. The feasible solution process could be heuristic or based on the approximate programming. In this paper, we adopt two steps to construct a feasible solution. First, based on the dual solution $\tilde{\boldsymbol{y}}_c$, we solve the SCUC problem (46) to obtain the power system schedule $\boldsymbol{x}^*$. In (46), $\boldsymbol{e}(\boldsymbol{x}_c)\le\boldsymbol{g}(\tilde{\boldsymbol{y}}_c)$ represents energy constraints or gas usage limits of gas-fired generating units. Then, we obtain a feasible solution $(\boldsymbol{x}^*,\boldsymbol{y}^*)$ by solving (47) based on the power system schedule $\boldsymbol{x}^*$.

$$\begin{aligned}&\boldsymbol{x}^*=\arg\min\mathrm{EC}(\boldsymbol{x})\\&\text{such that (4)–(16)}\\&\boldsymbol{e}(\boldsymbol{x}_c)\le\boldsymbol{g}(\tilde{\boldsymbol{y}}_c)\end{aligned} \tag{45}$$

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$$y^* = \arg\min \mathrm{GC}(y)$$
$$\text{such that (17)–(31)} \qquad (46)$$
$$g(y_c) = e(x_c^*)$$

4 Case studies

In the 6-bus power system and the 7-node gas system, we mainly study the impact of gas storage and the network congestion on the coordinated scheduling results. The 118 bus with 14-nodes shows the impact of price incentives on the calculation of the least social cost and the coordinated schedule. The comparisons of augmented LR and LR methods are given in both cases.

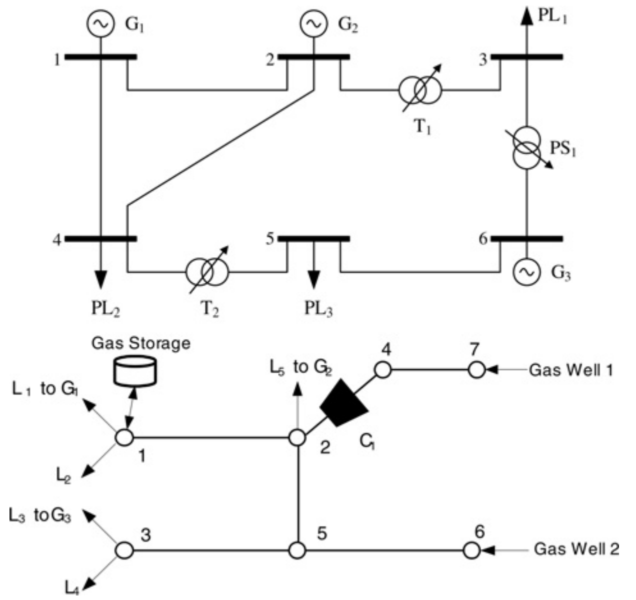

Figure 3 *Six-bus power system and 7-node natural gas system*

Table 1 Parameters of gas well

Gas well	Node no.	Well-head price, $/kcf	Min output, kcf/h	Max output, kcf/h
1	7	5.6	2000	5300
2	6	6	1000	6000

4.1 Six-bus power system and 7-node natural gas system

The 6-bus system and the 7-node natural gas system are depicted in Fig. 3. Parameters of the coupled power system and the natural gas system can be found in [8]. The cost information for the gas well and storage is given in Tables 1 and 2. Penalty prices for the electricity and the natural gas load not served are large as listed in Table 3. We apply the augmented LR and the LR methods to solve the following three cases.

Case 1: Base case without network or gas storage constraints

Case 2: Include network constraints

Case 3: Include network constraints and gas storage.

The augmented LR method is first applied to solve the three cases listed above. In Case 1, we ignore both electricity and natural gas transmission network constraints. Hence, there will be no congestion when scheduling the electricity and the natural gas systems. The hourly commitment of three gas-fired generating units is shown in Table 4. Fig. 4 shows the schedule of the gas well 2, which is the marginal gas well because of its higher well-head price. The social cost is $1 322 422, which is the lowest of all three cases. In Case 2, we consider the electricity and the natural gas transmission network constraints. The flow limits on power transmission line and the pressure limits on gas nodes through pipelines will result in the commitment of expensive generating units at additional hours. In comparison with those in Table 4, Table 5 shows

Table 2 Parameters of gas storage in Case 3

Storage node no.	Gas injection cost, $/kcf	Min. input, kcf/h	Max. input, kcf/h	Min. output, kcf/h	Max. output, kcf/h
1	2.5	150	2500	0	2000

Table 3 Electricity and gas load not served penalty price

penalty price electricity load not served ($/MWh)	2000
penalty price of gas load not served ($/kcf)	100

Table 4 Hourly commitments of Case 1 based on augmented LR

Unit	Hours (0–24)																								
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0
3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

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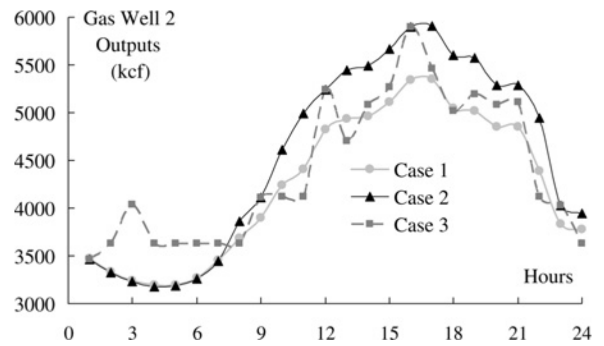

Figure 4 *Gas well 2 in Cases 1, 2 and 3 based on augmented LR*

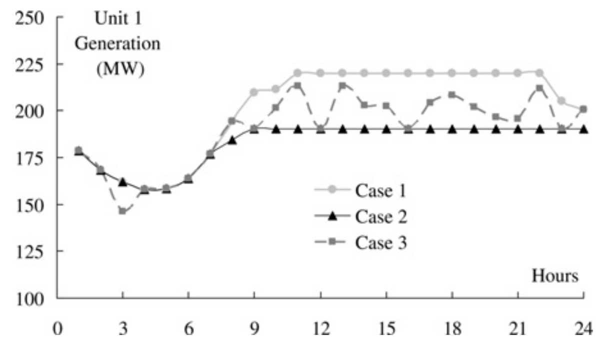

Figure 5 *Unit 1 dispatches in Cases 1, 2 and 3 based on augmented LR*

that the unit 2 is committed at hours 10–11, 22, and the unit 3 is on at hours 8–9 and 23–24. Fig. 5 shows that the most efficient unit G1 would generate less in Case 2. The social cost is $1 366 196, which is higher than that in Case 1. In Case 3, the simulation includes a gas storage, which draws natural gas from the pipeline into the gas reservoir at off-peak hours 2–7, and releases the natural gas during peak hours 8–24. The figure shows that by using the stored gas, the efficient generation unit 1 will generate more during hours 1–24. Table 6 shows the hourly schedule of generation units. The social cost is $1 350 932, which includes the cost of gas well and the operation of gas storage.

We also solve the three cases by LR in order to compare the performances of the two methods. Table 7 shows the results. As presented in Section 3, using the LR algorithm will cause oscillations in the dual solution. A feasible solution based on the dual solution of LR by (45) will lead to electricity load shedding. The augmented LR algorithm, on the other hand, can avoid such oscillations and result in a better solution. To further illustrate the worst convergence of the dual problem by LR, the violation of constraints (32) against iterations is plotted in Fig. 6. The values of LR and augmented LR are shown in Fig. 7. Here, the violation of (32) is defined by the Manhattan norm $\|\boldsymbol{e}(\boldsymbol{x}_{\mathrm{c}}) - \boldsymbol{g}(\boldsymbol{y}_{\mathrm{c}})\|_1$ of $\boldsymbol{e}(\boldsymbol{x}_{\mathrm{c}}) - \boldsymbol{g}(\boldsymbol{y}_{\mathrm{c}})$. Obviously, the constraint violations cannot be mitigated in the LR method. However, the violations will approach zero by increasing the number of iterations in the augmented LR.

4.2 118-bus power system and 14-node natural gas system

The modified IEEE 118-bus power system has 54 fossil units, 12 gas-fired combined cycle units, 7 hydro units and 91 demand sides. The natural gas transmission system is composed of 14 nodes, 12 pipelines and 2 compressors. The electricity and natural gas transmission system data are found in motor.ece.iit.edu/data/Gastranssmion_118_14test.xls. Table 8 lists the well-head prices of natural gas as well as the penalty price of residual natural gas loads not served. Table 9 shows the social costs.

Case 1: Base case solution: We solve the coordinated scheduling problem as formulated in this paper to obtain the least social cost schedules for coupled power and natural gas system. The daily social cost based on augmented LR in this case is $2 350 957. Table 10 shows the daily generation and resource information. In addition, Fig. 8 shows the hourly generation for comparison. The congestion occurs in the 24-h gas transmission. The joint operators supply the residual gas loads 1–5 fully because of their higher penalty prices. However, the gas consumption of gas-fired units and residual gas loads 6–8 with lower priority are curtailed through optimisation iterations.

Case 2: Impact of penalty price of residual gas loads: We assume the penalty price of residual gas loads not served in Case 2 is decreased as shown in Table 10. The results show that residual gas loads can be interrupted or not supplied with a lower social cost as compared to Case 1. When considering the coordinated social cost, the joint operators would prefer to supply the additional natural gas to gas-fired units for power generation rather than supplying the residual gas loads. In other words, the social benefits of supplying gas-fired units are higher than providing the natural gas to residual gas loads. As shown in Table 10, the MWh generation fuelled by natural gas in Case 2 is higher than that in Case 1. The natural gas supplied to residual gas loads is reduced from 229 126 to 148 015 kcf in

Table 5 Hourly commitments of Case 2 based on augmented LR

Unit	Hours (0–24)																								
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	0	0	0	0	0	0	0	*1*	*1*	1	1	1	1	1	1	1	1	1	1	*1*	0	0
3	0	0	0	*0*	*0*	*0*	*0*	*0*	*1*	*1*	1	1	1	1	1	1	1	1	1	1	1	1	1	*1*	*1*

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Table 6 Hourly commitments of Case 3 based on augmented LR

Unit	Hours (0–24)																							
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	0	0	0	0	0	0	0	*0*	*0*	1	1	1	1	1	1	1	1	1	1	*0*	0	0
3	0	0	0	*1*	*0*	*0*	*0*	*0*	*0*	*1*	1	1	1	1	1	1	1	1	1	1	1	1	1	*1*	*0*

Table 7 Comparison of augmented LR and standard LR based results

	Case index	Case 1	Case 2	Case 3
Augmented LR	dual cost ($)	1 322 408	1 366 196	1 350 931
	violation degree (kcf)	2.3	3.1	3.6
	feasible social cost ($)	1 322 422	1 366 196	1 350 932
	electricity load not served (MWh)	0	0	0
	gas load not served (kcf)	0	0	0
LR	dual cost ($)	1 319 070	1 343 842	1 344 346
	violation degree (kcf)	14 642	12 162	12 491
	feasible cost ($)	1 525 986	2 0743 44	1 935 640
	electricity load not served (MWh)	76.2	396.4	319.2
	gas load not served (kcf)	0	0	0

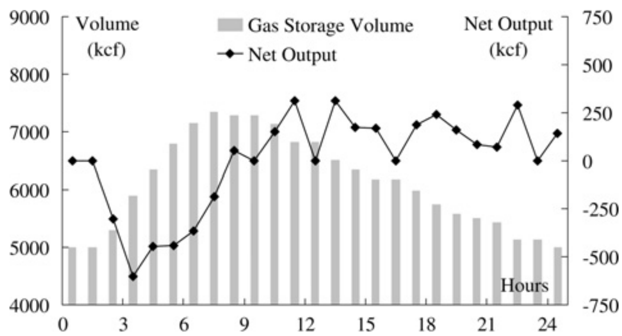

Figure 6 *Gas storage volume and output based on augmented LR*

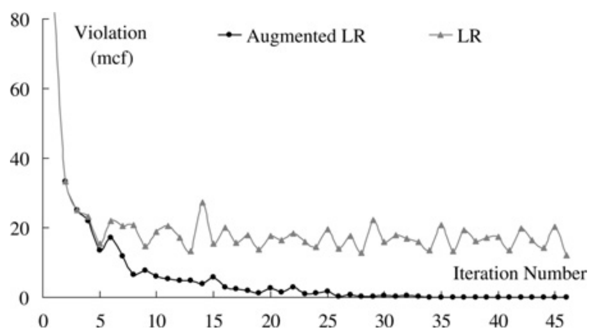

Figure 7 *Violation of dual iterations in Case 2*

Table 8 Well-head prices and gas load price incentives in Cases 1–3

Price incentives	Case 1	Case 2	Case 3
gas well 1	0.95	0.95	1.66
gas well 2	0.90	0.90	1.58
gas well 3	1.00	1.00	1.75
penalty price of residual gas loads 1–3 not served	3.00	1.20	3.00
penalty price of residual gas loads 4–5 not served	2.50	1.10	2.50
penalty price of residual gas loads 6–8 not served	1.10	0.90	1.80

Case 1. The social cost in Case 2 is $2 303 216. From Fig. 8, it is clear that the hourly coal generation in Case 1 is higher than that in Case 2.

Case 3: Impact of well-head price of natural gas: We increase the well-head price of natural gas wells by around 75% in comparison with that in Case 1 and solve the coordination scheduling problem by augmented LR. Table 10 shows that the power generation by natural gas in Case 1 is

Table 9 Social costs based on LT and augmented LR in Cases 1–3

Daily resources	Case 1	Case 2	Case 3	
social cost ($)	augmented LR	2 350 957	2 303 216	2 710 834
	LR	2 424 892	2 384 547	2 823 916

Table 10 Summarised daily generation and resource based on augmented LR

Daily resources	Case 1	Case 2	Case 3
power generation by coal (MWh)	120 796	114 312	131 724
power generation by natural gas (MWh)	14 689	21 178	3783
hydro power generation (MWh)	8308	8302	8286
supplied gas to residual loads (kcf)	229 126	148 015	305 526
supplied gas to gas-fired units (kcf)	190 270	275 309	48 907
consumed gas by compressors (kcf)	6374	6472	6186

replaced largely by coal generation because of the soaring natural gas price. The natural gas is no longer an economic choice for electricity generation in comparison with coal. Fig. 8 shows that in Case 3, natural gas units generate less than Case 1 and Case 2. Joint operators can dispatch more natural gas to residual gas loads. The daily social cost in Case 3 is $27 10 834, which is much higher than that in Case 1.

The daily and hourly hydro generation quantities are close in the three cases. The differences arise because the optimisation process will coordinate water resources to generate more during peak load hours or avoid committing more coal or gas-fired units while satisfying the hydro reservoir constraints.

We also solve the base case and the other two cases by LR. The social costs are given in Table 9. We draw a conclusion that the LR algorithm would result in a more expensive solution, whereas the augmented LR algorithm can avoid oscillations and lead to less expensive solutions. The computing time for obtaining dual solution of the LR method in Case 1 is 485 s with 20 iterations, whereas that based on augmented LR is 330 s with 15 iterations. The augmented Lagrangian decomposition and coordination method avoids the solution oscillation difficulties and speeds up algorithm convergence, thus augmented LR can obtain the solution more quickly than LR.

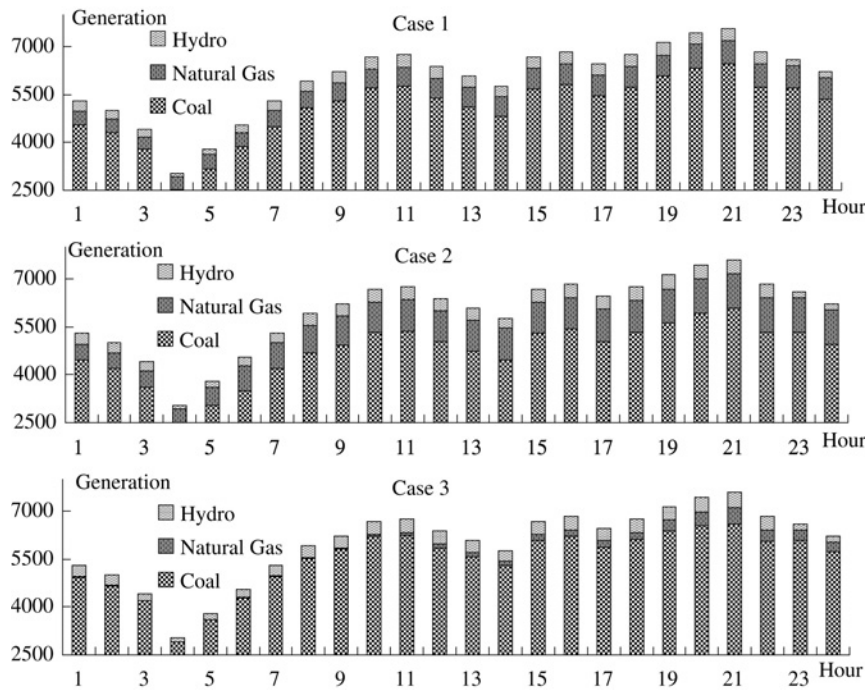

Figure 8 *Hourly generation composition based on augmented LR*

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5 Conclusions

This paper proposes a new model for the coordinated scheduling of the coupled electric power systems and natural gas transmission systems from a joint operator's viewpoint. The operator will coordinate the constrained resource scheduling and pursue the least social cost of energy system delivery. In this paper, the LR-based method is proposed initially to solve the problem. The augmented LR with the BDC technique is adopted to avoid numerical oscillations in this paper. Case studies verify that our proposed augmented LR method is effective in solving the coordinated energy scheduling model. Moreover, the augmented LR can avoid oscillations and improve the quality of the dual solution.

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