Contingency-Constrained PMU Placement in Power Networks

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***Abstract*—In this paper, a model for the optimal placement of contingency-constrained phasor measurement units (PMUs) in electric power networks is presented. The conventional complete observability of power networks is first formulated and then, different contingency conditions in power networks including measurement losses and line outages are added to the main model. The communication constraints which would limit the maximum number of measurements associated with each installed PMU is considered as measurement limitations. The relevant formulations are also proposed to make the model more comprehensive. The IEEE standard test systems are examined for the applicability of proposed model. The comparison of presented results with those of other methods is presented which would justify the effectiveness of proposed model with regards to minimizing the total number of PMUs and the execution time. A large-scale system with 2383 buses is also analyzed to exhibit the applicability of proposed model to practical power system cases.**

***Index Terms*—Contingency, integer linear programming, phasor measurement unit (PMU), transmission security.**

NOMENCLATURE

a_{ij}	Binary connectivity parameter between buses i and j.
a_{ij}^k	Binary connectivity parameter between buses i and j when line k is out.
f_i	Observability function of bus i.
f_i^k	Observability function of bus i when line k is out.
i, j	Indices of bus.
I	Set of buses.
k	Index of line.
K	Set of lines.
u_j	Binary decision variable that is equal to 1 if PMU is installed at bus j and 0 otherwise.
w_{ij}	Binary measurement variable of bus j related to the observability of bus i.
$w_j^{\max}$	Measurement limitation of bus j.
y_{ij}	Auxiliary binary variable of buses i and j.
y_{ij}^k	Auxiliary binary variable of buses i and j when line k is out.
z_j	Zero-injection parameter of bus j.

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I. INTRODUCTION

PHASOR measurement units (PMUs) were introduced into electric power systems based on additional developments in global positioning systems (GPS), communication networks, and digital signal processing techniques [1]. PMU is a device for synchronizing ac voltage and current measurements with a common time reference. The most common time reference is the GPS signal which has an accuracy of less than 1 μs [2]. In this way, the ac quantities are measured, converted to phasors (i.e., complex numbers represented by magnitude and phase angle), and time stamped [3]. Phasors at different nodes, which refer to the same time space coordinates, can improve the performance of monitored control systems in various fields of modern power systems. Some of these fields include state estimation, bad data detection, stability and control, remedial actions, and outage monitoring [4]–[7].

The first step in state estimation is to gather measurement data from substations. These measurements must be sufficient to make the system observable [8]. It is neither economical nor necessary to install a PMU at every node of a wide-area interconnected network. Hence, the problem is to find the minimum number of PMUs for the complete observability of network.

The optimal PMU placement problem is inherently an NP-complete problem with a solution space of 2^{N} possible combinations for an N-bus electric power system [9]. Therefore, the optimal PMU placement is considered as a combinatorial optimization problem and valuable research has been conducted in this area. The previously reported approaches are categorized into two groups: the meta-heuristic optimization methods and the conventional deterministic techniques. The meta-heuristic methods, which are based on the stochastic intelligent search process, do not require cost function derivatives and thus can deal with discrete variables and non-continuous cost functions. As the optimal PMU placement problem variables are discrete, these techniques have been widely applied to solve the PMU placement problem. Examples include canonical genetic algorithm [9], non-dominated sorting genetic algorithm [10], simulated annealing [11], Tabu search [12], simulated annealing combined with Tabu search [13], particle swarm optimization [14], and immunity genetic algorithm [15]. In [16], a binary search method was employed to solve the optimal PMU placement problem. The most critical obstacle for applying these methods is the large execution time which restricts their applications to large practical power systems. Another difficulty is that these methods do not offer any insight concerning the proximity of the current PMU placement solution to the global optimal.

There are considerable works on deterministic approaches to the optimal PMU placement problem [17]–[21]. In [17], the integer programming approach is applied to the PMU placement problem. The proposed model is new and useful; however, it introduces approximations when considering the effect of zero-injection buses. An algorithm using integer linear programming is proposed in [18]. The method takes into account power networks with and without conventional power flow and injection measurements. However, the proposed model does not consider the effect of zero-injection buses. The model presented in [18] was extended in [19] to consider the zero-injection effect, incomplete observability, and measurement redundancy. Reference [20] proposed another formulation based on integer linear programming in which the effect of zero-injection buses was incorporated. A multistage scheduling framework for PMU placement in a given time horizon was also proposed in [20]. The PMU placement problem using the integer quadratic programming was discussed in [21]. However, the effect of zero-injection buses was ignored in the model.

Among the reported approaches, few considered power system contingencies. Such contingencies comprise losses of measurements and/or line outages. The incorporation of contingencies in the optimal PMU placement problem would result in more reliable results. References [22]–[24] addressed the optimal placement methods with contingencies for conventional measurement devices. In [10], the model found observable network solutions when taking into account single line contingencies. In [25], a heuristic approach was presented to obtain a reliable PMU solution while retaining the network observability during single measurement losses and line contingencies. The presented model, which was based on numerical observability analyses, was computationally expensive. Reference [16] considered single line outages while the proposed exhaustive search was time consuming. In [20], PMU outages were considered; however, line outage contingencies were not incorporated. The outages of single lines and PMUs were considered in [21].

In this paper, we focus on a new model for the optimal contingency-constrained PMU placement. Initially, the basic model for the conventional observability of power networks is developed. Then, power network contingencies such as measurement losses and line outages are added to the basic model for enhancing its flexibility. The proposed model considers no approximations. In addition, measurement limitations representing power network communication constraints are considered. These limitations restrict the maximum number of PMUs. It is demonstrated that the proposed solution results are global optimal. The effectiveness of the proposed model is examined by comparing the results with those of other methods.

The remaining sections of this paper are organized as follows. Section II presents the proposed formulations of this paper. In this section, the basic model is introduced and different restrictions and constraints of power systems are added. The features of the proposed model are illustrated by a nine-bus network. In Section III, numerical results are presented for IEEE standard test systems as well as a large power system. Concluding remarks are discussed in Section IV.

II. Proposed PMU Formulation

The objective of PMU placement problem is to find the minimum number of PMUs as well as their placement to make the power network topologically observable. The problem is formulated as follows:

$$\text{Minimize} \sum_{j \in I} u_j \tag{1}$$

Subject to

$$f_i \geq 1, \ \forall i \in I \tag{2}$$

where

$$f_i = \sum_{j \in I} a_{ij} u_j, \quad \forall i \in I. \tag{3}$$

The objective function represents PMU installations which can be extended to consider PMU installation costs. In such a case, u_j will be replaced by $c_j u_j$ where c_j is the installation cost at bus j. However, employing $c_j u_j$ in the objective function has no effect on the linear format of proposed model.

In (3), f_i is the observability function at bus i. The binary connectivity parameter in (3) is defined as

$$a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if buses } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The observability of a bus depends on the installation of PMU at that bus or one of its incident buses. When all buses are observable, the value of observability function in (2) is equal to or greater than 1.

The following discussions will expand the above model by incorporating specific characteristics of power networks. The proposed options may be taken into account individually or simultaneously. In each case, a nine-bus system with nine lines and three zero-injection buses is analyzed.

A. *Effect of Zero-Injection Buses*

The zero-injection bus rules for assessing the network observability are:

1) When buses, which are incident to an observable zero-injection bus, are all observable except one, the unobservable bus will also be identified as observable by applying the KCL at zero-injection bus.
2) When buses incident to an unobservable zero-injection bus are all observable, the zero-injection bus will also be identified as observable by applying the KCL at zero-injection bus.

These two conditions can be combined into one by indicating that among a zero-injection bus and its incident buses, a single bus can be made observable by making the others observable. Using this simplification, the proposed formulation considering the effect of zero-injection buses is presented as

$$f_i \geq 1, \quad \forall i \in I \tag{5}$$

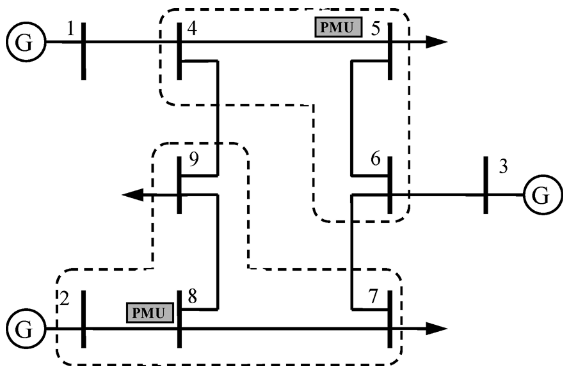

Fig. 1. Observability in the base case.

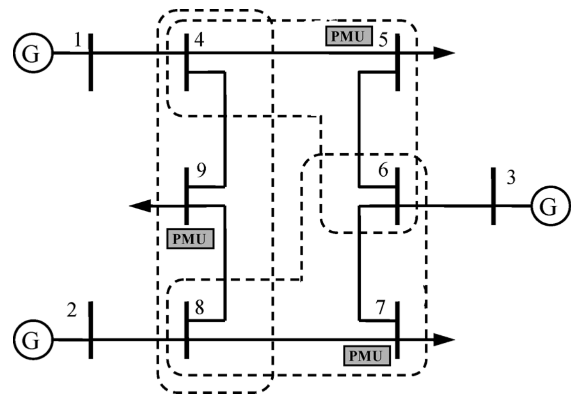

Fig. 2. Observability with no PMUs at zero-injection buses.

where

$$f_i = \sum_{j \in I} a_{ij} u_j + \sum_{j \in I} a_{ij} z_j y_{ij}, \quad \forall i \in I \tag{6}$$

$$\sum_{i \in I} a_{ij} y_{ij} = z_j, \quad \forall j \in I. \tag{7}$$

Expression (6) is the same as (3) with auxiliary binary variables added to zero-injection buses and those incident to zero-injection buses. Parameter z_j is a binary parameter that is equal to 1 if bus j is a zero-injection bus and 0 otherwise. For each nonzero-injection bus, the number of auxiliary variables is equal to the number of zero-injection buses which are incident to that bus. For each zero-injection bus, the number of auxiliary variables is equal to the number of zero-injection buses which are incident to that bus plus one. Obviously, all zero-injection buses would have at least one auxiliary variable.

When bus j is a zero-injection bus, the right hand side of (7) is equal to one. Therefore, exactly one auxiliary variable of buses which are incident to bus j or the auxiliary variable of bus j, would be equal to 1. When bus j is a nonzero-injection bus, the right hand side of (7) is zero. So all auxiliary variables of buses which are incident to bus j would be equal to zero. In fact, (6) and (7) simultaneously ensure that one of the buses which are incident to a zero-injection bus, or the zero-injection bus itself, will be observable when the others buses are observable.

The application of the proposed model to the nine-bus network results in the installation of two PMUs at buses 5 and 8 as shown in Fig. 1. In this case, buses 4, 6, and 8 are zero-injection buses and dashed lines show the observability zone of each installed PMU. Accordingly, buses 4 and 6 are made observable by the installation of PMU at bus 5, and buses 2, 7, and 9 are made observable by the installation of PMU at bus 8. Buses 1 and 3 are made observable through zero-injection effect of buses 4 and 6, respectively. Here, all buses have at least one source of observability; so the network is entirely observable.

B. No PMU at Zero-Injection Buses

The removal of PMUs from zero-injection buses will reduce the search space which could enhance the solution speed. In addition, PMUs at zero-injection buses measure current phasors of corresponding lines; thus, the KCL at zero-injection bus provides no additional information. However, the PMU placement at zero-injection buses will help find the optimal solution. Further explanations are provided in Section III.

The lack of PMUs at zero-injection buses is enforced by adding the following constraint:

$$z_j u_j = 0, \quad \forall j \in I. \tag{8}$$

Constraint (8) states that in zero-injection buses, where z_j is equal to 1, the value of u_j would be zero when no PMU is placed at those buses. At other buses, where z_j is zero, the value of u_j could be either zero or one.

In the previous example, one of PMUs was placed at bus 8 which is a zero-injection bus. If no PMUs are considered at zero-injection buses, the model finds three PMUs at buses 5, 7, and 9 for observability. Fig. 2 shows the PMUs and their associated observability zones. According to this figure, buses 4, 6, and 8 are made observable and each has one redundant measurement. Besides, buses 1, 2, and 3 are made observable through the zero-injection effect of buses 4, 8, and 6, respectively.

C. Loss of Measurement Contingency

In the proposed model, the loss of a PMU can be modeled by modifying the inequality (2). For representing the loss of single measurement, (2) is replaced by

$$f_i + \sum_{j \in I} a_{ij} y_{ij} \geq 2, \quad \forall i \in I. \tag{9}$$

Constraint (9) indicates that all buses will remain observable when a single PMU is lost. Here, $\sum_{j \in I} a_{ij} y_{ij} = 0$ when bus i is not observable by considering zero-injection buses. Accordingly, $f_i \geq 2$ which indicates that bus i would need a minimum of two observability sources. However, $\sum_{j \in I} a_{ij} y_{ij} = 1$ states that bus i is observable by a zero-injection bus. Since buses that affect zero-injections are observable during any PMU outages, the observability of bus i would be robust for any loss of PMUs. In Fig. 3, the proposed model would consider four PMUs in buses 4, 5, 7, and 8 for any single measurement loss. If any of the PMUs is lost, the buses will still be observable by considering the three other PMUs. Here buses 4, 5, 6, 7, 8, and 9 are observable when considering two PMUs locally or at adjacent buses. Bus 3 is observable through the zero-injection effect of bus 6. Since buses 5, 6, and 7 are observable during any PMU outages, bus 3 will also be observable during any such conditions. Similar deductions show that the observability of buses 1 and 2 are robust against any single PMU outages. Furthermore, these buses are observable when considering PMUs at buses 4 and 8, respectively.

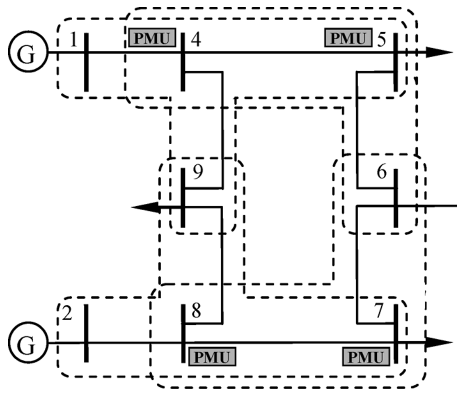

Fig. 3. Observability considering the loss of a single measurement.

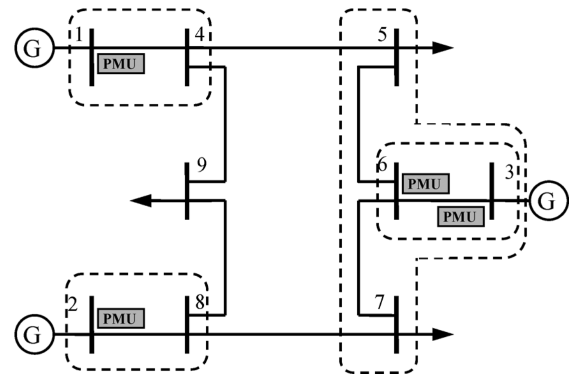

Fig. 4. Observability considering a single line outage.

D. Line Outage Contingency

The effect of a single line outage is added to the proposed model using the following set of constraints:

$$f_i^k \geq 1, \quad \forall i \in I, \forall k \in K \tag{10}$$

where

$$f_i^k = \sum_{j \in I} a_{ij}^k u_j + \sum_{j \in I} a_{ij}^k z_j y_{ij}^k, \;\; \forall i \in I, \forall k \in K \tag{11}$$

$$\sum_{i \in I} a_{ij}^k y_{ij}^k = z_j, \quad \forall j \in I, \forall k \in K. \tag{12}$$

Expressions (10)–(12) are the same as (5)–(7), respectively, while the connectivity parameters, auxiliary variables, and observability functions are replaced with those representing the outage of line k. All expressions are repeated over $k \in K$ to consider a line outage contingency. The binary connectivity parameter when line k is out is defined as follows:

$$a_{ij}^k = \begin{cases} 0, & \text{if line } k \text{ is between buses } i \text{ and } j \\ a_{ij}, & \text{otherwise.} \end{cases} \tag{13}$$

More line outages may be considered similarly in the proposed model. The modeling of a single line outage in the nine-bus network results in the placement of four PMUs at buses 1, 2, 3, and 6. These four PMUs maintain the network observability during any single line outages.

Fig. 4 shows the observability zone of installed PMUs in which the bus observability analysis is performed as follows: Buses 1, 2, 3, and 6 are observable using their associated installed PMUs. Bus 4 is made observable by the PMU installed at bus 1; however, when line 1–4 is on outage, bus 4 is made observable by the zero-injection effect. Bus 5 is made observable by the PMU installed at bus 6 which has redundant measurements by the zero-injection of buses 4 and 6. When line 5–6 is on outage, it is made observable by zero-injection of bus 4. Similar to bus 5, bus 7 is made observable by the PMU installed at bus 6. However, when line 6–7 is on outage, it is made observable by the zero-injection of bus 8. Bus 8 is made observable by the PMU installed at bus 2 when line 2–8 is in service. Otherwise, bus 8 is made observable by its zero-injection effect. Bus 9 is made observable by the zero-injection of buses 4 and 8. Therefore, its observability remains in effect when either line 4–9 or line 8–9 is on outage.

E. Measurement Limitations

The process of communicating PMU measured data to a control center requires an extensive set of communication equipment. Usually multi-channel PMUs are installed at buses with several measurements [9], [26]. We consider the effect of limited communication on PMU placements by substituting $\sum a_{ij} u_j$ with $\sum a_{ij} w_{ij} u_j$ in (3) and (6). Hence

$$f_i = \sum_{j \in I} a_{ij} w_{ij} u_j, \quad \forall i \in I. \tag{14}$$

In addition, the following constraints are included:

$$\sum_{i \in I} a_{ij} w_{ij} \leq w_j^{\max}, \quad \forall j \in I \tag{15}$$

$$w_{ij} \leq u_j, \quad \forall i, j \in I. \tag{16}$$

The binary variable w_{ij} represents the measurement at bus i using a PMU placed at bus j. Hence, (15) is considered as another constraint in order to limit the number of measurements associated with bus j. Inequality (16) states that the placement of a PMU allows the related binary measurement variables to be either zero or one; however, associated binary measurement variables are zero when a PMU is not installed. Here, the nonlinear expression $w_{ij} u_j$ is the product of two binary variables and the following constraints are used to convert the nonlinear expression to linear:

$$r_{ij} = w_{ij} u_j \tag{17}$$

$$r_{ij} \leq u_j \tag{18}$$

$$r_{ij} \leq w_{ij} \tag{19}$$

$$r_{ij} \geq u_j + w_{ij} - 1. \tag{20}$$

In (18)–(20), the nonlinear variable r_{ij}, which is defined in (17), is expressed as a set of three linear inequalities with binary variables. Furthermore, (18) can be deleted when (16) and (19) are introduced.

The nine-bus network is examined assuming that each PMU has at most two measurements. Hence, a PMU placed at a bus measures its own voltage phasor and one current phasor associated with the lines incident to that bus. The results presented in Fig. 5 consist of three PMUs at buses 5, 7, and 9. Here, buses 4, 6, and 8 are observable directly and buses 1, 2, and 3 are observable by the zero-injection of buses 4, 8, and 6, respectively. Comparing Figs. 5 and 2, we learn that the observability zone of each PMU is restricted by limited measurements.

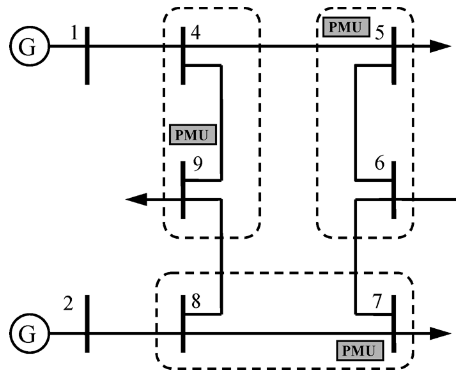

Fig. 5. Observability considering measurement limitations.

TABLE I
OBSERVABILITY ANALYSIS OF NINE-BUS NETWORK IN DIFFERENT STATES

Bus No.	Base Case State	No PMU at Zero-Injection	Loss of Measurement	Measurement Limit
1	Z4	Z4	P4, Z4	Z4
2	P8, Z8	Z8	P8, Z8	Z8
3	Z6	Z6	Z6	Z6
4	P5	P5, P9	P4, P5	P9
5	P5	P5	P4, P5	P5
6	P5	P5, P7	P5, P7	P5
7	P8	P7	P7, P8	P7
8	P8	P7, P9	P7, P8	P7
9	P8	P9	P4, P8	P9

Table I summarizes the observability analysis of different states, i.e., normal, no PMU at zero-injection buses, loss of measurement, and measurement limitation. Table II shows the observability details associated with a line outage contingency state. In these tables, Pi indicates that the corresponding bus is observable by the PMU placed at bus i, and Zi indicates that the corresponding bus is observable by the zero-injection effect of bus i. Tables I and II assess the conclusions derived from Figs. 1–5 which are summarized as follows:

- All buses have at least one source of observability.
- There are three sources of observability in each state when considering the effect of zero-injection buses, i.e., Z4, Z6, and Z8. This outcome is in accordance with the rules for zero-injection buses.
- In Table I, the column entitled Loss of Measurement indicates that all buses have redundant measurements and, when considering the loss of a single measurement, they will remain observable by other observability sources.
- In Table II, when considering any line outages, buses remain observable by means of other sources. This result ensures that the model performs well in the event of line outages.

III. CASE STUDIES

The performance of the proposed model is examined by applying the IEEE test systems as well as a large power system. All cases are implemented on a 1.86 GHz processor with 512 MB of RAM using CPLEX 9.0 solver [27] in the GAMS environment [28]. In CPLEX, an optimality gap can be specified to choose between an optimal solution and a quick suboptimal solution. This gap is defined as the absolute value of $100 \times (BI - BL)/BL$ where BI is the existing best integer solution and BL is the existing lower bound. Accordingly, if the gap goes to zero, the solution is global optimal.

TABLE II
OBSERVABILITY ANALYSIS OF NINE-BUS NETWORK CONSIDERING SINGLE LINE OUTAGE

Bus No.	Line Outage				
	Base Case	1-4	4-5	5-6	3-6
1	P1	P1	P1	P1	P1
2	P2, Z8	P2	P2	P2	P2
3	P3, P6	P3, P6	P3, P6	P3, P6	P3
4	P1	Z4	P1	P1	P1
5	P6, Z6	P6, Z6	P6, Z6	Z4	P6, Z4, Z6
6	P3, P6	P3, P6	P3, P6	P3, P6	P6
7	P6	P6	P6, Z8	P6, Z6	P6
8	P2	P2	P2	P2	P2
9	Z4	Z8	Z4	Z8	Z8

Bus No.	Line Outage				
	6-7	7-8	2-8	8-9	4-9
1	P1	P1	P1	P1	P1
2	P2	P2	P2	P2	P2
3	P3, P6	P3, P6	P3, P6	P3, P6	P3, P6
4	P1	P1	P1	P1	P1
5	P6, Z6	P6, Z4	P6, Z6	P6, Z6	P6, Z4
6	P3, P6	P3, P6	P3, P6	P3, P6	P3, P6
7	Z8	P6, Z6	P6	P6, Z8	P6, Z6
8	P2	P2	Z8	P2	P2
9	Z4	Z8	Z4	Z4	Z8

TABLE III
CHARACTERISTICS OF IEEE STANDARD TEST SYSTEMS

Test System	No. of Lines	No. of Zero-Injection Buses	Max. No. of Lines Connected to a Bus
IEEE 14-Bus	20	1	5
IEEE 30-Bus	41	6	7
IEEE 39-Bus	46	12	5
IEEE 57-Bus	80	15	6
IEEE 118-Bus	186	10	12

A. IEEE Standard Cases

First, the standard IEEE 14-, 30-, 39-, 57-, and 118-bus test systems are investigated by the proposed model. Table III tabulates the specific characteristics of these test systems.

Five different conditions are considered, i.e., base case state, no PMU at zero-injection buses, loss of measurement, line outage, and loss of measurement or line outage. The effect of zero-injection buses is incorporated in all cases. The minimum number of PMUs and their locations are shown in Tables IV

TABLE IV
RESULTS OF PMU PLACEMENT FOR THE IEEE STANDARD TEST SYSTEMS IN DIFFERENT STATES

Test System	Base Case State	No PMU at Zero-Injection	Line Outage	Loss of Measurement	Line Outage or Loss of Measurement
IEEE 14-Bus	3	3	7	7	8
IEEE 30-Bus	7	7	13	15	17
IEEE 39-Bus	8	8	15	18	22
IEEE 57-Bus	11	11	19	26	26
IEEE 118-Bus	28	28	53	63	65

TABLE V
PMU PLACEMENT BUSES FOR THE IEEE STANDARD TEST SYSTEMS

Test System	Base Case State	No PMU at Zero-Injection	Line Outage	Loss of Measurement	Line Outage or Loss of Measurement
IEEE 14-Bus	2, 6, 9	2, 6, 9	1, 3, 6, 8, 9, 11, 13	2, 4, 5, 6, 9, 10, 13	1, 2, 4, 6, 8, 9, 10, 13
IEEE 30-Bus	3, 5, 10, 12, 18, 24, 27	1, 2, 10, 12, 18, 24, 29	1, 3, 5, 10, 11, 13, 14, 15, 16, 19, 23, 26, 30	1, 3, 5, 7, 10, 12, 13, 15, 16, 19, 20, 24, 25, 27, 29	1, 3, 5, 7, 10, 11, 12, 13, 15, 16, 19, 20, 23, 24, 26, 27, 30
IEEE 39-Bus	3, 8, 11, 16, 20, 23, 25, 29	3, 8, 12, 16, 20, 23, 25, 29	3, 8, 16, 24, 26, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38	3, 6, 8, 9, 12, 14, 16, 18, 20, 21, 23, 25, 26, 29, 34, 36, 37, 38	3, 6, 8, 9, 10, 16, 18, 20, 21, 23, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38
IEEE 57-Bus	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	1, 2, 6, 12, 14, 19, 21, 27, 29, 30, 32, 33, 41, 44, 49, 51, 53, 55, 56	1, 2, 4, 6, 9, 12, 14, 19, 20, 24, 25, 27, 29, 30, 32, 33, 38, 39, 41, 44, 46, 50, 51, 53, 54, 56	1, 2, 4, 6, 9, 12, 14, 19, 20, 24, 25, 27, 29, 30, 32, 33, 36, 38, 41, 44, 46, 50, 51, 53, 54, 56
IEEE 118-Bus	3, 9, 11, 12, 17, 21, 25, 28, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114	3, 8, 11, 12, 17, 21, 25, 28, 34, 35, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114	1, 6, 10, 11, 12, 15, 17, 19, 21, 23, 26, 27, 29, 34, 35, 39, 41, 44, 46, 49, 51, 53, 56, 57, 59, 61, 67, 72, 73, 74, 75, 76, 78, 80, 83, 85, 87, 89, 91, 92, 94, 96, 100, 101, 105, 107, 109, 111, 112, 113, 114, 116, 117	1, 3, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 26, 27, 28, 29, 32, 34, 35, 37, 40, 41, 43, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 66, 68, 70, 71, 72, 75, 76, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117	1, 3, 5, 7, 8, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 28, 29, 32, 34, 35, 37, 40, 41, 44, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 66, 68, 72, 73, 74, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 107, 109, 110, 111, 112, 115, 116, 117

and V, respectively. In these cases, the PMU placement at zero-injection buses makes no difference. In essence, the model finds identical numbers of PMUs in both cases but at different locations. In all cases, as expected, the line outage contingency case would require more PMUs than the base case and the loss of measurement would require more PMUs than the line outage contingency case.

Table IV shows that the contingency case with a loss of line or measurement would require the highest number of PMUs. In this state, more than half of buses are equipped with PMUs in order to obtain a reliable measurement system and accurate state estimation.

The case with PMU measurement limitations is analyzed here. It is assumed that all PMUs are the same with similar measurement limitations. Ten different cases are examined, where measurements are limited to 1 through 10. Table VI shows that when all PMUs have one measurement, the number of required PMUs is equal to the number of network buses minus the number of zero-injection buses. This result is obvious because the total number of buses equal to the number of zero-injection buses is made observable by zero-injection effect and the others would require PMUs. Although the assumption of one measurement for each PMU is impractical, it helps confirm the ability and accuracy of the model in the worst case. Table VI shows that if measurement limitations are removed, which is equivalent to the utilization of more communication facilities, the required number of PMUs will be lower. Tables III and VI show that the required number of PMUs will not change when the measurement limitation exceeds the maximum number of lines connected to a bus.

Table VI shows the minimum number of measurements for PMUs when calculating the optimal number of PMUs. This concept is used in the planning and the installation of communication equipment since the corresponding costs could be excessively high.

The optimality gap was set to zero which indicates that the results are global optimal. Another interesting point about these results is that the execution time associated with each case is less than 5 s. The quality of proposed solutions is assessed in Table VII by comparison with those derived by the previously reported methods. The table shows the superiority of the proposed model based on the minimum number of PMUs.

TABLE VI
RESULTS OF PMU PLACEMENT FOR THE IEEE STANDARD TEST SYSTEMS CONSIDERING MEASUREMENT LIMITATION

Test System	Maximum Number of Measurements									
	1	2	3	4	5	6	7	8	9	10
IEEE 14-Bus	13	7	5	4	3	3	3	3	3	3
IEEE 30-Bus	24	12	8	7	7	7	7	7	7	7
IEEE 39-Bus	27	14	9	8	8	8	8	8	8	8
IEEE 57-Bus	42	21	14	12	11	11	11	11	11	11
IEEE 118-Bus	108	54	36	30	28	28	28	28	28	28

TABLE VII
COMPARISON OF TOTAL NUMBER OF PMUS OBTAINED BY SEVERAL METHODS

Test System	Integer Programming [17]	Genetic Algorithm [9]	Nondominated Sorting Genetic Algorithm [10]	Simulated Annealing [11]	Tabu Search [12]	Particle Swarm Optimization [14]	Binary Search [16]	Immunity Genetic Algorithm [15]	Proposed Model
IEEE 14-Bus	3	3	-	3	3	3	3	3	3
IEEE 30-Bus	-	7	-	7	-	7	7	7	7
IEEE 39-Bus	-	-	8	-	10	-	8	-	8
IEEE 57-Bus	12	12	-	11	13	11	-	11	11
IEEE 118-Bus	29	29	29	-	-	28	-	28	28

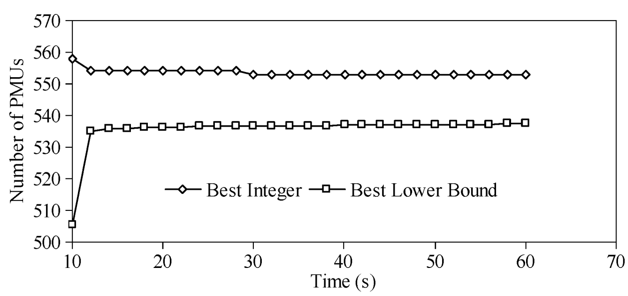

Fig. 6. Evolution of the best solution and best lower bound in base case.

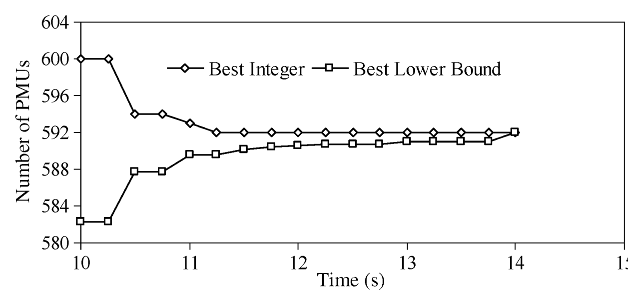

Fig. 7. Evolution of the best solution and best lower bound in no PMU case.

B. Large Power System

In order to investigate the effectiveness of proposed approach in large-scale power systems, the 2383-bus polish power system which includes 2896 lines and 552 zero-injection buses is studied [29]. The computational performance is assessed for the base case and the case with no PMU placements at zero-injection buses. In the base case, the CPLEX solver starts by solving the relaxed problem and allowing all integer variables to be temporarily continuous. This solution takes 10 seconds which establishes an initial lower bound on the objective function. The solver then proceeds to the cutting plane logic as well as Branch and Bound as shown in Fig. 6. The figure shows the results over the first 60 s. The PMUs are 554 in the 12th second and 553 in the 28th second. The solver terminates the process with a best integer solution of 553 PMUs and the optimality gap of 2%.

For the no PMU placement at zero-injection buses, the CPLEX presolve process takes 10 s which establishes an initial lower bound of 582 PMUs on the objective function. The solver then proceeds to the cutting plane logic along with Branch and Bound. Fig. 7 shows the computing time for the best solution as well as the lower bound. In this figure, after 14 s the best solution and the lower bound converge to 592. The solver would accordingly terminate the process as the optimal solution is found with a zero optimality gap.

TABLE VIII
COMPARISON OF COMPUTATIONAL DIMENSION

State	Base Case	No PMU at Zero-Injection
No. of single equations	5319	5871
No. of single variables	7025	7025
No. of decision variables	2383	1831
No. of discrete variables	4641	4641

The good quality of near-optimal solution in base case and the optimal solution in the no PMU placement case at zero-injection buses justifies the practical applicability of the proposed model. The comparison of Figs. 6 and 7 shows that the base case begins the search from a better initial solution which results in fewer PMUs; the no PMU case converges faster and finds a global optimal solution.

Table VIII compares the two simulation cases. The assumption that no PMU is placed at zero-injection buses will increase the number of single equations in the model by the number of zero-injection buses (i.e., $5871 - 5319 = 552$). These equations will set to zero the binary decision variable of zero-injection buses. So the number of decision variables decreases by the number of zero-injection buses (i.e., $2383 - 1831 = 552$). The search space will be reduced to half by eliminating a single candidate PMU. So the reduction in the number of decision variables has a significant impact on the search space and, consequently, on the execution time. However, the difference between best solutions in these two cases shows that in large power systems the assumption of no PMU at zero-injection buses may not be beneficial to the optimality of solution.

IV. Conclusions

In this paper, a fast and practical model based on integer linear programming is proposed for solving the optimal PMU placement problem. Different contingency conditions associated with power systems, i.e., line outages and loss of measurements were considered. The additional conditions can be considered separately or simultaneously in practical power systems. This capability makes the proposed model more flexible as compared to existing models. In addition, communication constraints of power networks were considered as measurement limitations and included in the model. The proposed model is successfully tested on the IEEE standard test systems in addition to a large-scale system. For the IEEE standard test systems, the minimum number of PMUs was compared with those of existing methods. The salient features of the proposed method include a low execution time as well as global optimality that make the method suitable for large-scale power system applications.

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