Reliability Modeling of PMUs Using Fuzzy Sets

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***Abstract*—Probabilistic analyses of a wide-area measurement system (WAMS) would require equivalent reliability models for its components which include phasor measurement units (PMUs). In this paper, the reliability modeling of PMU is proposed and the proposed model is extended to consider options for the PMU hardware. The Markov process is employed to analyze the proposed model and to present an equivalent two-state model of PMUs. Reliability parameters of PMU are estimated with major difficulties associated with limited and uncertain data. In this paper, uncertainties are taken into account to achieve more realistic estimates of PMU characteristics. Fuzzy sets along with reliability analyses and fuzzy importance measures (FIMs) are utilized as a means of measuring the significance of PMU components on its availability. Numerical analyses are conducted and results are discussed.**

***Index Terms*—Fuzzy sets, phasor measurement units, reliability modeling.**

Nomenclature

A_{bc}^{CT}	Availability of the CT adjacent to bus b associated with line bc.
A_b^{link}	Availability of the communication link corresponding to PMU at bus b.
A_b^{PMU}	Availability of PMU at bus b.
$A^\alpha(x)$	α-cut of the fuzzy set A.
a_1^α, a_2^α	Lower and upper limits of $A^\alpha(\cdot)$, respectively.
b, c	Indices for bus.
$ED(\cdot,\cdot)$	Euclidean distance between two fuzzy numbers.
FIM_i	Fuzzy importance measure of component i.
i	Index for state and component.
$m_A(x)$	Membership degree of x in fuzzy set A.
p_{DN}	Probability of residing at the down state in the PMU's equivalent model.
p_i	Probability of residing at state i in the PMU seven-state model.
p_{UP}	Probability of residing at the up state in the PMU's equivalent model (availability).
U_i	Unavailability of component i.
λ_e	Failure rate of the PMU's equivalent model, (failure/year).
λ_i	Failure rate of component i (failure/year).
μ_e	Repair rate of the PMU's equivalent model (repair/year).
μ_i	Repair rate of component i (repair/year).
λ_i^1, μ_i^1	Lower bounds of fuzzy failure and repair rates, respectively (occurrence/year).
λ_i^2, μ_i^2	Kernels of fuzzy failure and repair rates, respectively (occurrence/year).
λ_i^3, μ_i^3	Upper bounds of fuzzy failure and repair rates, (occurrence/year).

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I. Introduction

THE extensive utilization of power networks, catastrophic outages, and complicated market-driven operations have heightened the requirements for novel monitoring and control algorithms in power systems [1]. The wide-area measurement system (WAMS) has evolved into a practical tool for such monitoring applications [2]. The key element of WAMS is the synchronized measurement technology which has been enabled by the commercial development of phasor measurement units (PMUs). The synchronization of PMUs is via signals from global positioning system (GPS) [3]. Advanced applications of WAMS offer a cost-effective solution to improve the system operation, control, modeling, and energy trading [1], [4]–[8]. Accordingly, the role of WAMS and the success of its operation have become extremely vital. Any failures even in a small portion of WAMS could result in unobservable operating conditions and consequently threaten the security of power systems. So, reliability analyses and probabilistic studies of WAMS are essential endeavors along with its progressive design and applications.

The reliability assessment of WAMS would require the identification of its components, e.g., PMUs, and their characteristics and functions, and the derivation of analytical models to represent such characteristics. Reference [9] discussed the basic designs and special applications of WAMS and probed the availability of WAMS in simple applications. For each application, the required number of PMUs was calculated for the partial or complete observability of the system and the corresponding availability of WAMS was computed. The paper assumed a typical value for the availability of PMUs.

The reliability data for a device can be determined either by applying statistical methods to historical data or using the reliability modeling of the device as a single system. In the case of PMUs, there is not a rich historical database available since the device is newly developed. So this paper offers a reliability model for PMUs. A reliability model for PMU was proposed in [10] in which reliability transition rates were considered as crisp numbers. In reliability evaluation methods, input parameters such as failure and repair rates were derived from historical records which could be subject to errors. In addition, these parameters exhibit significant unit-to-unit variability because of the stochastic nature of failure conditions, and environment changes. The literature shows that the failure data can deviate from the norm by a factor of 3 or 4, and a factor of 10 is not unusual either [11]. Hence, the true and single-point values of these parameters are not given which could lead to errors in reliability evaluations. Two conventional approaches that would incorporate such uncertainties include assuming a probability distribution for input parameters and using either the conditional probability method [12] or the Monte Carlo simulation [13]. However in most cases, it is difficult to find proper probability distributions. In addition, the conventional probabilistic methods could be computationally cumbersome and might require enormous computing resources for large systems. The methodologies based on fuzzy sets [14] can explicitly consider uncertainties of input parameters in reliability analyses. They could also produce possibility distributions instead of a single-point output. Furthermore, such applications could consider a subjective set of information quantified via expert opinions [15]–[17].

This paper considers the reliability modeling of PMUs using a Markov process. PMU components and their functions are first described and a state space reliability model is developed. The proposed model is analyzed to offer an equivalent two-state reliability model for PMUs and its availability is subsequently calculated based on the proposed model. PMUs design and characteristics will vary depending on a specific manufacturer. Here, a few PMU structures are presented along with their reliability models to make the proposed model more flexible. Fuzzy sets are considered for the reliability analysis when incorporating the effect of uncertainties associated with input data. Numerical examples are examined to investigate the effectiveness of the proposed model. The proposed PMU reliability model could be used for the probabilistic assessment of WAMS. PMU manufacturers may also utilize the proposed model for identifying the critical PMU components and as such optimize investments in PMU components to improve its availability.

The remaining sections of this paper are organized as follows. The PMU structure and its functional block diagram are described in Section II. Reliability modeling of PMU and fuzzification of analysis are respectively presented in Sections III and IV. Section V conducts numerical studies on the proposed model and conclusions drawn from the paper are discussed in Section VI.

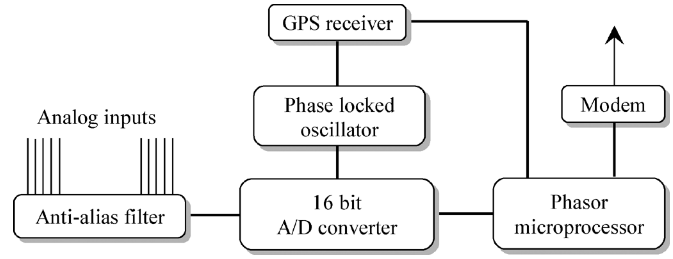

Fig. 1. Block diagram of PMU.

II. PHASOR MEASUREMENT UNIT (PMU)

Fig. 1 shows the hardware block diagram of PMU [3]. Analog inputs consist of three-phase values of all required voltages and currents measured respectively by potential transformers (PTs) and current transformers (CTs) installed in substations. Note that these measuring transformers are not considered as PMU hardware elements. Anti-aliasing filter is used to filter out from the input waveform frequencies above the Nyquist rate. Analog measured inputs are converted into digital signals via a 16 bit A/D converter. Phase locked oscillator converts the GPS' one pulse per second into a sequence of high-speed timing pulses used in the waveform sampling. Phasor microprocessor executes phasor calculations and phasors are finally time stamped and transmitted by means of modem. Obviously, PMU components should be supplied by a direct current source. So, the power supply is another element of PMU which is not depicted in Fig. 1.

Given the complexity in the power system signal environment, good filtering is required in the actual PMU logic, though there are several options available for filtering [18], [19]. Fig. 1 is a more common PMU structure.

III. RELIABILITY MODELING OF PMU

Reliability modeling and evaluation techniques [20], [21] are employed to develop a reliability model for PMU. Each component in Fig. 1 can reside in either up or down state. If none of the components have redundancy, the failure of any component would result in the system failure. In this case, components are called in series from a reliability point of view. Fig. 2 illustrates the state space representation of this model. State 0 is the working state of PMU and State $i, i = 1, \ldots, 6$, is the down state of PMU caused by the component i failure. It is assumed that when the PMU is at its failure state, no subsequent component failures can occur until the PMU returns to its operating state. Component failures are assumed independent and the power supply is fully reliable. The steady-state solution based on the frequency balance approach is as follows:

$$p_0 \cdot \lambda_i = p_i \cdot \mu_i; \quad i = 1, \ldots, 6. \tag{1}$$

Therefore

$$p_i = \frac{\lambda_i}{\mu_i} p_0; \quad i = 1, \ldots, 6. \tag{2}$$

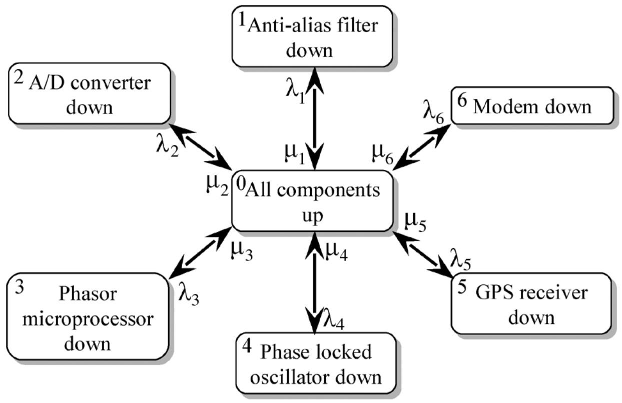

Fig. 2. Seven-state Markov model of PMU.

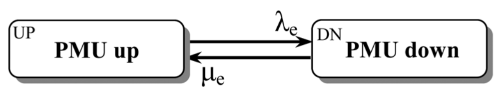

Fig. 3. Equivalent two-state model of PMU.

By substituting p_i in $p_0 + \sum_{i=1}^{6} p_i = 1$, we have

$$p_o = \left(1 + \sum_{i=1}^{6} \frac{\lambda_i}{\mu_i}\right)^{-1}. \tag{3}$$

The probabilities associated with other states can be obtained by (2).

As States 1–6 of the Markov model would result in the PMU failure, they can be merged into one down state. This reduction results in an equivalent two-state model shown in Fig. 3. The parameters of this equivalent model are given as

$$p_{UP} = p_0 = \left(1 + \sum_{i=1}^{6} \frac{\lambda_i}{\mu_i}\right)^{-1} \tag{4}$$

$$p_{DN} = 1 - p_{UP} = \left(\sum_{i=1}^{6} \frac{\lambda_i}{\mu_i}\right) \cdot \left(1 + \sum_{i=1}^{6} \frac{\lambda_i}{\mu_i}\right)^{-1} \tag{5}$$

$$\lambda_e = \sum_{i=1}^{6} \lambda_i \tag{6}$$

$$\mu_e = \left(\sum_{i=1}^{6} \lambda_i\right) \cdot \left(\sum_{i=1}^{6} \frac{\lambda_i}{\mu_i}\right)^{-1}. \tag{7}$$

The extension of the proposed model to incorporate additional elements are discussed here.

A. Additional PMU Components

The hardware structures could be specific to PMU manufacturers. The reliability model in Fig. 2 is specific to the given structure. The extendibility of the model is considered in the proposed reliability model. Engineering judgment of practitioners is crucial in this stage of the modeling procedure. For instance, from the signal quality viewpoint, an output filter might be installed between phasor microprocessor and modem [19]. If the successful operation of PMU is assumed to be dependent on the availability of the output filter, the filter should be added to the state space. However, the output filter might be necessary only for a specific set of PMU applications. In such a case, the PMU should be modeled separately for each application and if the operation of PMU does not depend on the availability of the filter, the filter should be omitted from the reliability model.

To incorporate redundant components of PMUs in the state space model, each group of redundant components are replaced by a two-state component and this equivalent is then considered in the state space model. Obviously, calculations are similar to (4)–(7). As an example, assume that the PMU has two redundant phasor microprocessors. In general, reliability parameters associated with these microprocessors can be different. Assuming λ_3', μ_3' and λ_3'', μ_3'' as transition rates associated with microprocessors 1 and 2, respectively, the failure and repair rates associated with the equivalent two-state model is calculated as [20]

$$\lambda_3 = \lambda_3' \cdot \lambda_3'' \cdot \left(\frac{1}{\mu_3'} + \frac{1}{\mu_3''}\right) \tag{8}$$

$$\mu_3 = \mu_3' + \mu_3''. \tag{9}$$

B. Software Reliability

PMUs use software algorithms to carry out certain functions such as the computation of phasors magnitude and angle, calculation of symmetrical component, estimation of frequency and change of frequency, determination of harmonic content, etc. Obviously, the software might include programming bugs and so work improperly in some situations. Such anomalies can cause PMU failures which must be appropriately modeled in reliability evaluations. Software reliability methods determine the reliability of a software by applying statistical inference techniques to software failure data.

Several software reliability models are proposed in [22]. However, the model proposed by Shooman is a commonly used technique [23]. Once the reliability transition rates associated with the software are calculated, they should be taken into account in the PMU reliability evaluation. Software errors could result in PMU failures. Hence, from the reliability viewpoint, the software is in series with other hardware components and an additional state for the software is considered in Fig. 2. Another possible alternative to incorporate the software in the PMU reliability modeling is to use the series system approximation technique to combine transition rates associated with the microprocessor hardware and software [20].

IV. Fuzzy-Based Reliability Analysis

We utilize fuzzy sets to incorporate inherent uncertainties of PMU parameters, i.e. failure and repair rates, in reliability calculations. Accordingly, reliability parameters in Fig. 2 are modeled as fuzzy numbers [24], [25]. The wider the support of the membership function, the higher is the uncertainty [26].

The computational efficiency is a very important issue associated with fuzzy analyses. Kaufman and Gupta showed that the computation for fuzzy analyses can be reduced by composing membership functions into α-cuts and conducting mathematical operations on these intervals [27]. The crisp set of elements that belong to a fuzzy set at least to the degree α is called α-cut

TABLE I
COMPONENTS RELIABILITY DATA

Component	λ_i (Failure/year)			μ_i (Repair/year)		
	λ_i^1	λ_i^2	λ_i^3	μ_i^1	μ_i^2	μ_i^3
Anti-alias filter	0.03	0.15	0.45	219	438	657
A/D converter	0.10	0.15	0.45	219	438	657
Phasor microprocessor	0.15	0.40	1.60	438	876	1314
Phase locked oscillator	0.08	0.15	0.60	219	438	657
GPS receiver	0.40	0.60	2.40	438	876	1314
Modem	0.45	0.65	1.95	438	876	1314

of that fuzzy set. Assume the fuzzy number A is defined on X. The α-cut of this fuzzy set is

$$A^{\alpha}(x) = [a_1^{\alpha}, a_2^{\alpha}] = \{x | m_A(x) \geq \alpha\}\,; \alpha \in [0,1]. \quad (10)$$

The following steps describe the fuzzy reliability analysis of PMU.

Step 1) Represent the input data, i.e. failure and repair rates, by fuzzy numbers. Membership functions are determined based on the historical data or expert's opinion.

Step 2) Establish α-cuts of input data for $\alpha \in [0,1]$.

Step 3) Calculate parameters of the equivalent reliability model for any α, using set of (4)–(7) and fuzzy arithmetic operations [24]. For each α-cut of the fuzzy number which represents a parameter, the calculation of (4)–(7) is fulfilled to determine the minimum and maximum possible values of the output. If the output is monotonic with respect to the dependent fuzzy inputs, the process is rather simple since only two simulations will be enough for each α-cut. Otherwise, optimization routines are required to determine the minimum and maximum values of the output for each α-cut [28]. The former is the case here, since Markov process approach is used for the reliability modeling and equations derived for the final outputs are explicitly defined in terms of inputs as well.

Step 4) The α-cut outputs calculated in Step 3) are used to construct fuzzy outputs.

Each fuzzy output is associated with a possibility degree which provides an appropriate insight on the distribution of results. If the user needs to express the results by crisp numbers, defuzzification methods such as center of area (CA) or mean of maximum should be employed [24]. CA method is utilized in this paper. It is worth noting that although the defuzzified result is only a single-tone value (similar to the crisp result), it incorporates the effects of uncertainties associated with input data.

A. Fuzzy Importance Measure (FIM)

One significant quantity in the reliability assessment is the measures of importance which provides the sensitivity of results with respect to component failures. A larger measure of importance indicates that the corresponding component has more impact on results. Measures of importance in an uncertain environment considering the fuzzy parameters can be achieved by FIM [29].

FIM is explained via a simple example. Suppose that A is the fuzzy number representing p_{UP}. A can be implicitly expressed as a function of component unavailabilities, i.e. $A = f(U_1, \ldots, U_6)$. Assume that $A_{U_i=1}$ and $A_{U_i=0}$ are associated with A. To calculate $A_{U_i=0}$, the failure rate of component i is assumed to be a single-tone zero and its repair rate be a reasonable non-zero number. However, to calculate $A_{U_i=1}$, the component i and its corresponding state are omitted from the reliability model. FIM_i is then computed as

$$FIM_i = ED\left(A_{U_i=1}, A_{U_i=0}\right). \quad (11)$$

The Euclidean distance for fuzzy sets A and B is calculated as

$$ED(A,B) = \sqrt{\sum_{\alpha \in [0,1]} \left[\left(a_1^{\alpha} - b_1^{\alpha}\right)^2 + \left(a_2^{\alpha} - b_2^{\alpha}\right)^2 \right]}. \quad (12)$$

FIMR, which is the FIM of redundancy, is defined to determine the application of PMU redundant components. Accordingly, the resulting parameters are computed in two different cases with and without redundant components. In the first case, A is calculated without redundancy of component i and in the second case, it is assumed that component i has a redundant component and the resulting parameter A_i^R is then calculated. FIMR would be the Euclidean distance between A and A_i^R. Obviously, the component with the largest FIMR is the first candidate to be redundantly configured.

V. Numerical Studies

In this section, several studies are presented to demonstrate the merits of the proposed reliability model. Both crisp and fuzzy models are examined, and FIM and FIMR calculations are presented for PMU components. Finally, a simple example will exhibit the application of the proposed model for the reliability evaluation of PMU applications.

In reliability studies, reliability transition rates are unsymmetrical and single-kernel fuzzy numbers. As shown in Table I, the kernel point of data is multiplied with a factor in this study to simulate fuzzy numbers. The data used for simulations lie within ranges associated with electronic components of digital relays [30]. Certain PMU components such as phasor microprocessor, GPS receiver, and modem are composed of integrated circuits with their own failure rates. However, transition rates are assumed here for the entire component.

One important point is that defective electronic components of PMU are usually replaced rather than repaired. Therefore, the repair rate corresponds to the replacement rate in such cases. Triangular fuzzy sets are used for transition rates unless otherwise is noted. All calculations use 101 α -cuts. In all figures and tables, units of failure and repair rates are respectively failure/year and repair/year.

A. Reliability Analyses

In order to examine the proposed reliability model, we consider the following four cases:

- Case 0: Crisp input data.

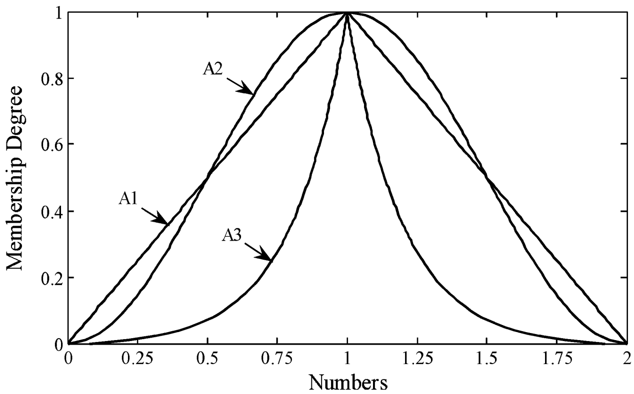

Fig. 4. Membership functions.

TABLE II
RESULTS OF CRISP RELIABILITY ANALYSIS

Parameter	Value	Parameter	Value
p_{UP}	0.997097	λ_e	2.10
p_{DN}	0.002903	μ_e	721.4

- Case 1: A1 represents membership functions.
- Case 2: A2 represents membership functions.
- Case 3: A3 represents membership functions.

Typical configurations for A1, A2, and A3 with a support set of [0,2] and kernel of 1 are depicted in Fig. 4.

Case 0: Equations (4)–(7) are used to calculate crisp reliability parameters for the equivalent two-state model of PMU. All transition rates are assumed to be equal to the kernel of corresponding fuzzy numbers, namely λ_i^2 and μ_i^2. The results for the equivalent model are given in Table II in which the PMU has a high availability rate. However, this rate does not promise a high availability rate for the whole WAMS. As an example, the IEEE 118-bus system would need at least 28 PMUs for the complete observability [31]. Assuming that other devices in the system are fully reliable, the probability of a fully observable system is $(p_{UP})^{28} = 0.921835$, which means that the system is unobservable about 648 hr/yr. This observation justifies the additional research on the probabilistic planning of PMU-based applications.

Case 1: The fuzzy calculation is employed for analyzing the equivalent two-state model. Table III presents 11 α-cuts for these parameters as depicted in Fig. 5. Comparing Tables II and III, we learn that the fuzzy results with $\alpha = 1$ are expectedly identical with crisp results. In Fig. 5, p_{UP}, p_{DN}, and λ_e are unsymmetrical while μ_e is approximately symmetrical as it is mostly dependent on μ_i which is symmetrical. Obviously, CA of p_{UP}, p_{DN}, and λ_e are not equal to their kernels which is caused by uncertainties in input data.

Cases 2 and 3: Cases 2 and 3 illustrate the solution sensitivity to membership functions. According to Fig. 4, the membership function can be adjusted for maximizing the utility. In Fig. 4, A3 represents a more precise definition for numbers close to 1 than those defined by A1 and A2. The results for Cases 2 and 3 are depicted in Fig. 5. Here, A3 results preserve the precision of input data as compared with those of A1 and A2. Table IV shows defuzzified results of Cases 1, 2, and 3.

TABLE III
RESULTS OF FUZZY RELIABILITY ANALYSIS

α	$p_{UP_1}^{\alpha}$	$p_{UP_2}^{\alpha}$	$p_{DN_1}^{\alpha}$	$p_{DN_2}^{\alpha}$
1	0.997097	0.997097	0.002903	0.002903
0.9	0.996181	0.997358	0.002642	0.003819
0.8	0.995166	0.997594	0.002406	0.004834
0.7	0.994033	0.997810	0.002190	0.005967
0.6	0.992761	0.998008	0.001992	0.007239
0.5	0.991324	0.998190	0.001810	0.008676
0.4	0.989687	0.998359	0.001641	0.010313
0.3	0.987804	0.998515	0.001485	0.012196
0.2	0.985617	0.998660	0.001340	0.014383
0.1	0.983045	0.998795	0.001205	0.016955
0	0.979975	0.998920	0.001080	0.020025
α	λ_{e1}^{α}	λ_{e2}^{α}	μ_{e1}^{α}	μ_{e2}^{α}
1	2.10	2.10	721.4	721.4
0.9	2.01	2.64	687.4	759.0
0.8	1.92	3.17	652.5	796.9
0.7	1.83	3.71	617.2	835.2
0.6	1.74	4.24	581.5	873.8
0.5	1.66	4.78	545.6	913.0
0.4	1.57	5.31	509.6	952.7
0.3	1.48	5.85	473.4	993.0
0.2	1.39	6.38	437.2	1034.2
0.1	1.30	6.92	400.9	1076.3
0	1.21	7.45	364.6	1119.7

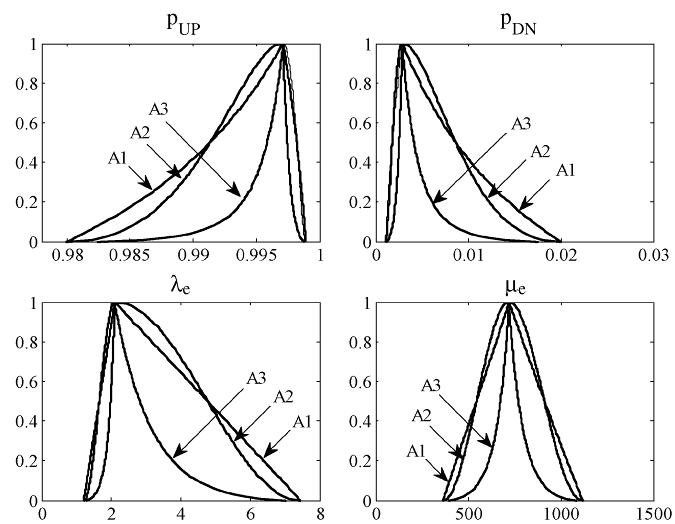

Fig. 5. Fuzzy results of the PMU equivalent two-state model.

TABLE IV
DEFUZZIFIED RESULTS OF CASES 1, 2, AND 3

Parameter	Value		
	Case 1	Case 2	Case 3
p_{UP}	0.995498	0.995382	0.996753
p_{DN}	0.004502	0.004618	0.003247
λ_e	2.84	2.94	2.31
μ_e	726.9	727.4	723.0

Here, the results of Cases 1 and 2 are close; however, Case 3 results are close to those of crisp analysis. Comparing Tables II and IV illustrates that for all membership functions, the PMU

TABLE V
CRISP RESULTS WITH NEW DATA

Parameter	Value	Parameter	Value
p_{UP}	0.996871	λ_e	2.30
p_{DN}	0.003129	μ_e	732.6

availability calculated by fuzzy analysis is less than that of crisp calculation because of uncertainties in the input data. The results for fuzzy analyses are more realistic than those of crisp study. Also we use triangular membership functions (A1) since its results incorporate the effect of uncertainties appropriately and its corresponding computations are also plain. Table IV shows that the repair rates of two-state equivalent model are similar in Cases 0 through 3 because the membership functions of repair rates in Fig. 5 are mostly symmetric.

B. Worth of Fuzzy Analyses

The crisp problem could be solved individually for all possible combinations of input data with uncertainties. However, the task would be computationally intractable for large systems while the application of fuzzy sets would provide a single solution representing all such cases. As an instance, consider that the input data are the same as those of Case 0, except for the failure rate of phasor microprocessor, $\lambda_3^2 = 0.60$, which is increased by 50%. The crisp solution in Table V shows that the availability of PMU is reduced as expected. Since $\lambda_3^2 = 0.60$ is within the range of fuzzy input data in Table I, the new crisp result lies in the fuzzy results range of [0.979975,0.998920]. This observation reveals that resultant fuzzy numbers include all possible results arising from variations of the input data.

C. Importance Measure Analyses

FIM is used to investigate the impact of components on the PMU availability. Here FIMs are close and within the range of [14.104431,14.117178] because the PMU components do not have any redundancy. Table VI presents the component ranking based on the FIM and importance measure (IM) calculated for the crisp reliability model. IM is calculated similar to FIM while assuming that all input data as single-tone fuzzy numbers. Table VI shows that anti-alias filter, A/D converter, and phase locked oscillator have identical IM from the crisp reliability viewpoint, while from the fuzzy reliability viewpoint phase locked oscillator has a bit more impact on the PMU availability compared to A/D converter. The reason for this is that the data uncertainty of phase locked oscillator is higher than that of A/D converter. In addition, Table VI shows different IM and FIM ranking for GPS receiver and modem. As given in Table I, the crisp failure rate of modem is greater than that of GPS receiver while their repair rates are the same. Therefore, it is reasonable that modem is recognized as being more important by IM. However, the failure rate of GPS receiver shows more uncertainty and is therefore admitted as more important than modem by FIM.

The following two approaches may be considered to increase the availability of PMUs: 1) Incorporate more reliable components which could be technically infeasible or expensive and 2) incorporate redundant components [22]. The latter approach is more commonly used. FIMR is defined in this paper to identify the PMU elements whose redundancy would have the highest impact on the system availability. The numerical results of FIMR indicate that the importance of component redundancy is similar to that of FIM given in Table VI. So, the most eligible component for redundancy is the GPS receiver which also has the highest impact on the PMU availability since it holds the highest FIM. However, the ranks derived by FIM and FIMR, in spite of their similarities, may not be identical. Considering $(\lambda_6^1, \lambda_6^2, \lambda_6^3) = (0.45, 0.95, 1.95)$ as the fuzzy failure rate of modem, the component rank order based on FIM becomes similar to that of IM in Table VI while the component rank order based on FIMR will not change. So the GPS receiver, which is the most eligible component to be configured redundantly, does not have the highest impact on the PMU availability.

TABLE VI
FIMS AND IMS OF PMU COMPONENTS

Component	Rank based on FIM	Rank based on IM
Anti-alias filter	6	4
A/D converter	5	4
Phasor microprocessor	3	3
Phase locked oscillator	4	4
GPS receiver	1	2
Modem	2	1

D. Applications of PMU Reliability Model

An example here shows how the information on the availability of PMU is utilized. Consider a transmission line with PMUs at both ends. Accordingly, a differential protection scheme is considered and the information on the availability of this protection scheme is of interest here. Differential protection is based on the comparison of current phasors at both ends of the transmission line and consequently in addition to PMUs, CTs and communication links are to be available. PMU would measure a phasor of current or voltage using symmetrical components [32] so the three phase measurement of CTs will be required. There is no direct communication link between any two PMUs which are connected to data concentrators as shown in Fig. 6 [3]. Data concentrators, super data concentrators, and communication links, are assumed fully reliable with redundant and backup elements. Accordingly, only communication links between PMU and data concentrator are considered in the reliability studies.

Hence, PMUs and CTs at both ends of a line as well as communication links should be available. This means that all these devices are in series and the availability of the protection system is therefore equal to the product of all their availabilities [20]. Mathematically speaking, the availability of differential protection system of the line connected between buses b and c will be

$$A^{DPS} = A_b^{PMU} \cdot A_c^{PMU} \cdot \left(A_{bc}^{CT}\right)^3 \cdot \left(A_{cb}^{CT}\right)^3 \cdot A_b^{link} \cdot A_c^{link}. \tag{13}$$

Assuming A_b^{PMU} and A_c^{PMU} are equal to the defuzzified p_{UP} associated with Case 1, $A_{bc}^{CT} = A_{cb}^{CT} = 0.999584$ [33], $A_b^{link} = A_c^{link} = 0.999$, and A^{DPS} will be 0.986571. Other examples are given in [9].

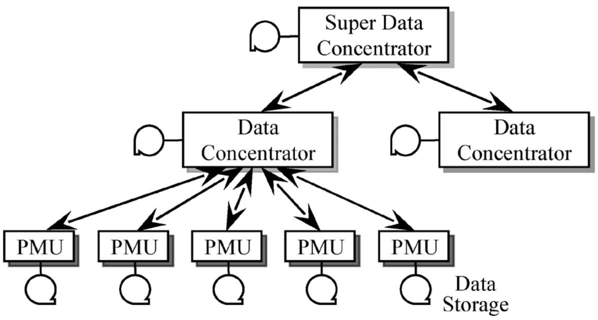

Fig. 6. Hierarchy of WAMS [3].

VI. Conclusions

The reliability modeling of PMU was considered here by applying the Markov process approach. Fuzzy analysis is used to take into account the effect of input parameter uncertainties. Fuzzy calculations are based on the effective α-cut method. The final equivalent two-state model is used to calculate probabilities and transition rates of the PMU reliability model. Numerical example was examined using both crisp and fuzzy data. Results comparison showed that if the crisp parameters are assumed to be equal to the kernel of fuzzy parameters, the resultant availability of PMU by fuzzy calculations is less than that of crisp analysis. This is due to uncertainties associated with input data. The importance of components from the reliability viewpoint has been done by calculating FIMs. FIMs show that, without having redundancy, all internal components have nearly the same influence on PMU unavailability. However, more fuzziness of a component's parameter would result in more FIM of that component. The availability of a transmission line differential protection system has been calculated based on the calculated availability of the PMU. This simple example was presented only to show the probabilistic studies of WAMS which are presently being considered by researchers.

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