Infinite-horizon Economic MPC for HVAC Systems with Active Thermal Energy Storage

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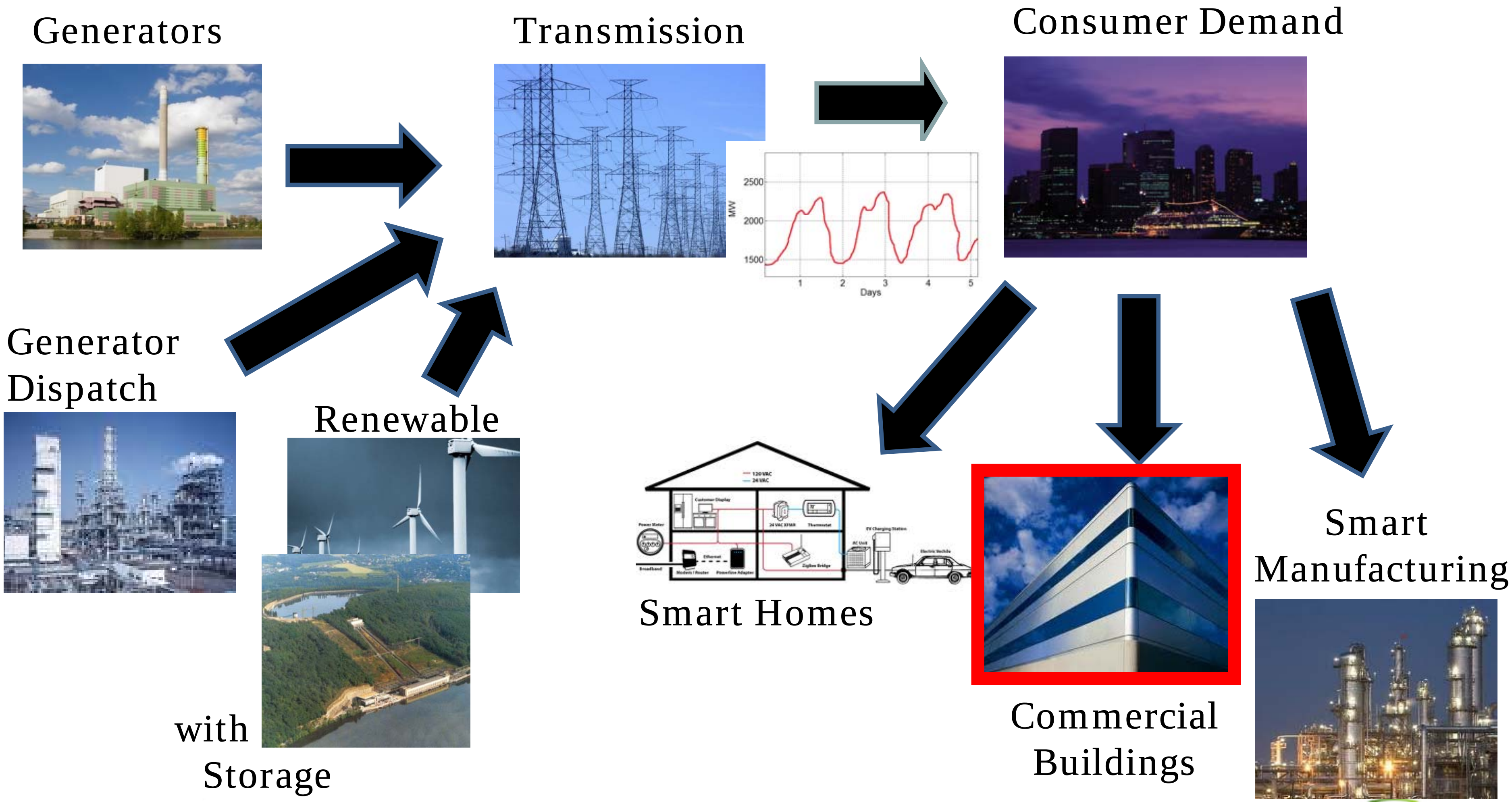
Overview of Smart Grid



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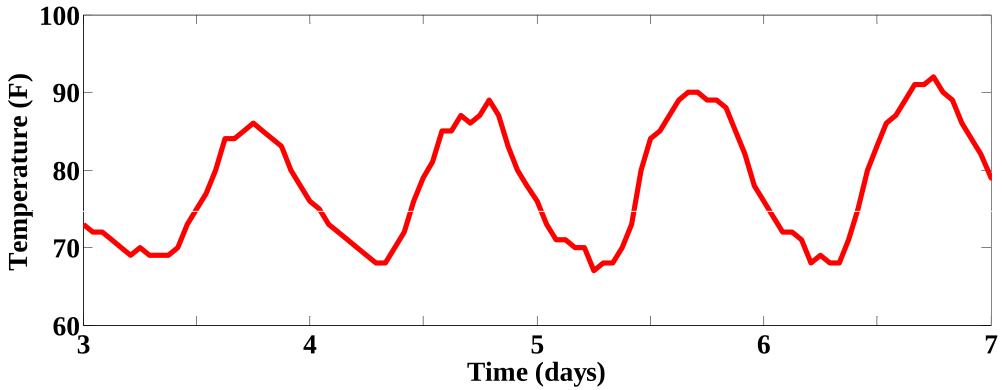
Presentation Outline

- Background
  - Building HVAC and TES
  - Economic Model Predictive Control (EMPC)
- Application of EMPC to Building HVAC with TES
  - Economic Benefits
  - Some Issues with EMPC
- Economic Linear Optimal Control (ELOC)
- Infinite Horizon EMPC (IH-EMPC)

Cooling Power Consumption

Cooling is mainly required during the hottest times of a day…



Outside Temperature.
August 3 - 6, 2001. Pittsburg, PA.

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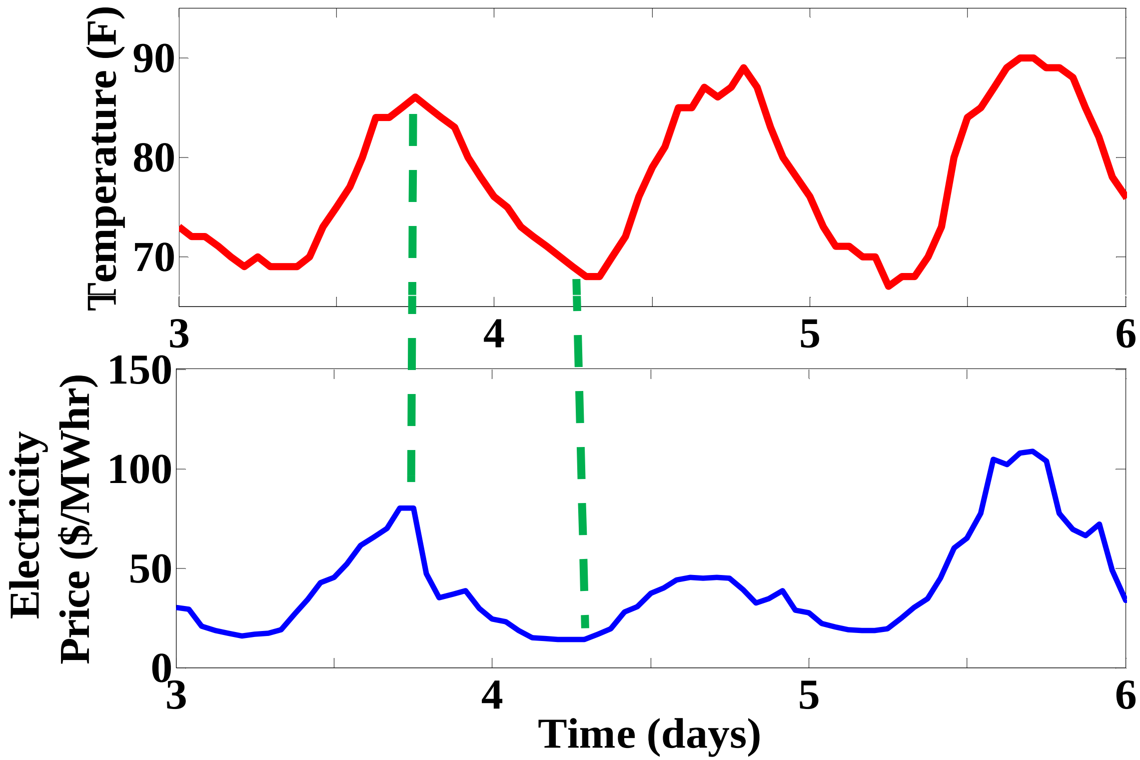
Correlation Between Cooling Loads and Energy Prices



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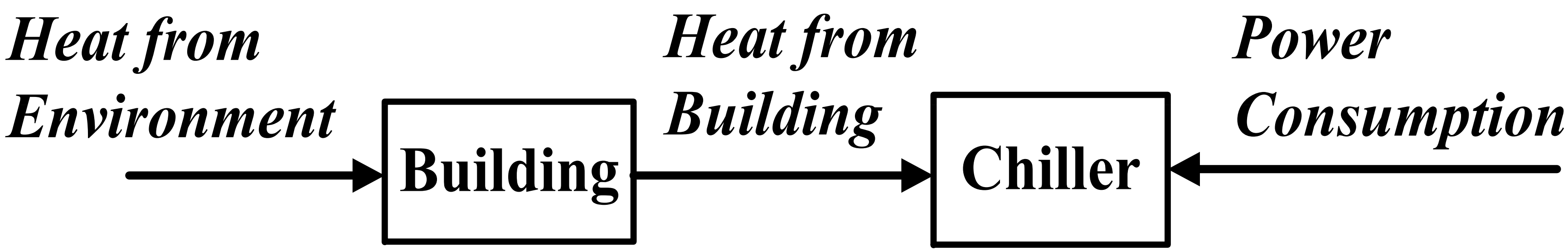
Traditional HVAC System

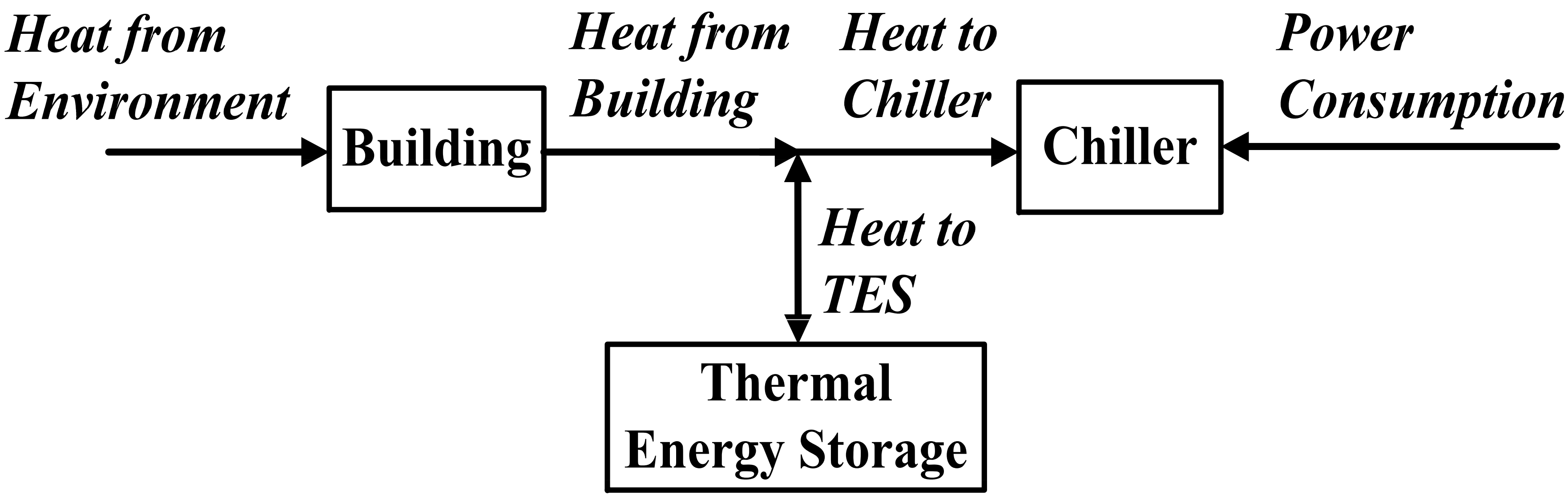


- Heat is removed from the building by a chiller
- Chiller consumes electric power
- Assume real-time prices for electricity

Thermal Energy Storage

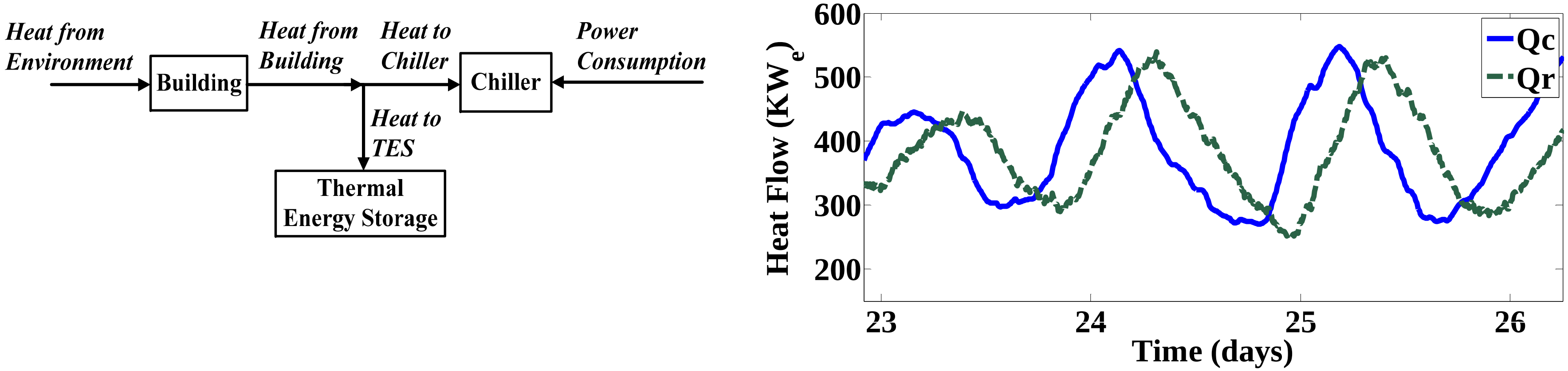
TES helps time shift electricity consumption to periods of low electricity prices.

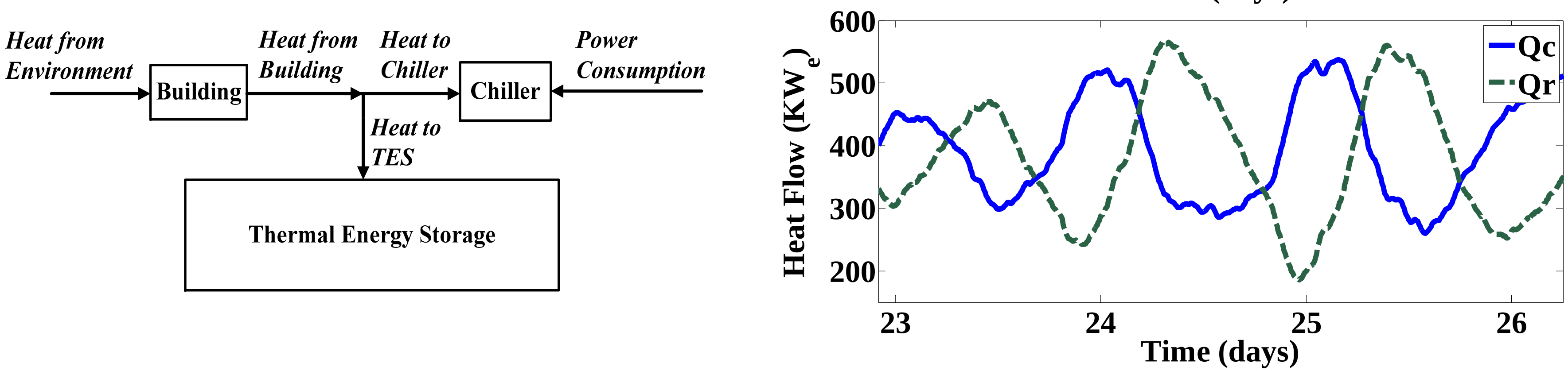
HVAC System with TES



- Building heat can be sent to chiller or TES
- Heat must eventually be removed from TES by chiller.

Impact of Thermal Energy Storage





Nonlinear Model Predictive Control

$$\min_{x,u,w} \int_{t}^{t+T} g\left(x,u,w\right)\, d\tau$$

$$\begin{aligned} s.t. \quad \dot{x} &= f\left(x,u,w\right) \\ z &= h(x,u,w) \\ z^{\min} \le z(\tau) &\le z^{\max} \end{aligned}$$

Traditional MPC Objective

Quadratic Objective

$$g\left(x,u,w\right)=x^{T}Qx+u^{T}Ru$$

$$\min_{x,u,w}\int_{t}^{t+T} g\left(x,u,w\right)\,d\tau$$

$$s.t.\quad \dot{x}=f\left(x,u,w\right)$$

$$z=h(x,u,w)$$

$$z^{\min}\leq z(\tau)\leq z^{\max}$$

Economic MPC

<u>Economic Objective</u>

$g\left(x,u,w\right)=$ Instantaneous Expenditures

$$\min_{x,u,w} \int_{t}^{t+T} g\left(x,u,w\right)\, d\tau$$

$$s.t. \quad \dot{x} = f\left(x,u,w\right)$$

$$z = h(x,u,w)$$

$$z^{\min} \le z(\tau) \le z^{\max}$$

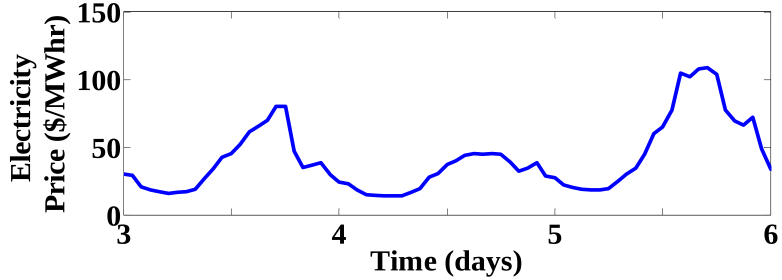
Literature on EMPC

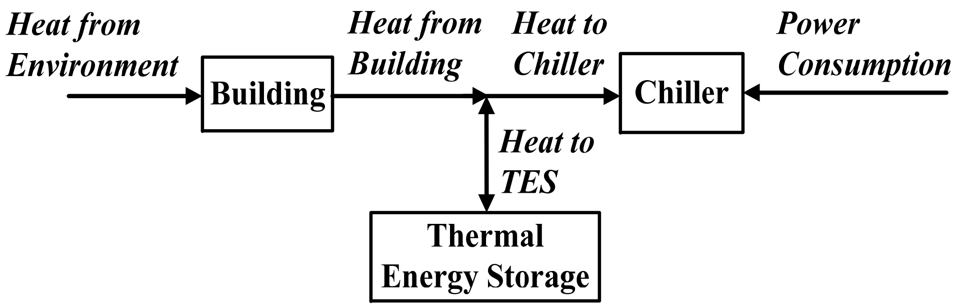
- **Conceptual Development and Stability Issues**: Rawlings and Amrit (2009); Diehl, et al. (2011); Huang and Biegler (2011); Heidarinejad, et al. (2012)
- **Process Scheduling**: Karwana and Keblisb (2007); Baumrucker and Biegler (2010); Lima et al. (2011); Kostina et al. (2011)
- **Power Systems**: Zavala et al. (2009); Xie and Ilić (2009), Hovgaard, et al. (2011), Omell and Chmielewski (2011)
- **HVAC Systems**: Braun (1992); Morris et al. (1994); Kintner-Meyer and Emery (1995); Henze et al. (2003); Braun (2007); Oldewurtel et al. (2010), Ma et al. (2012); Mendoza and Chmielewski (2012)

Economic MPC for HVAC

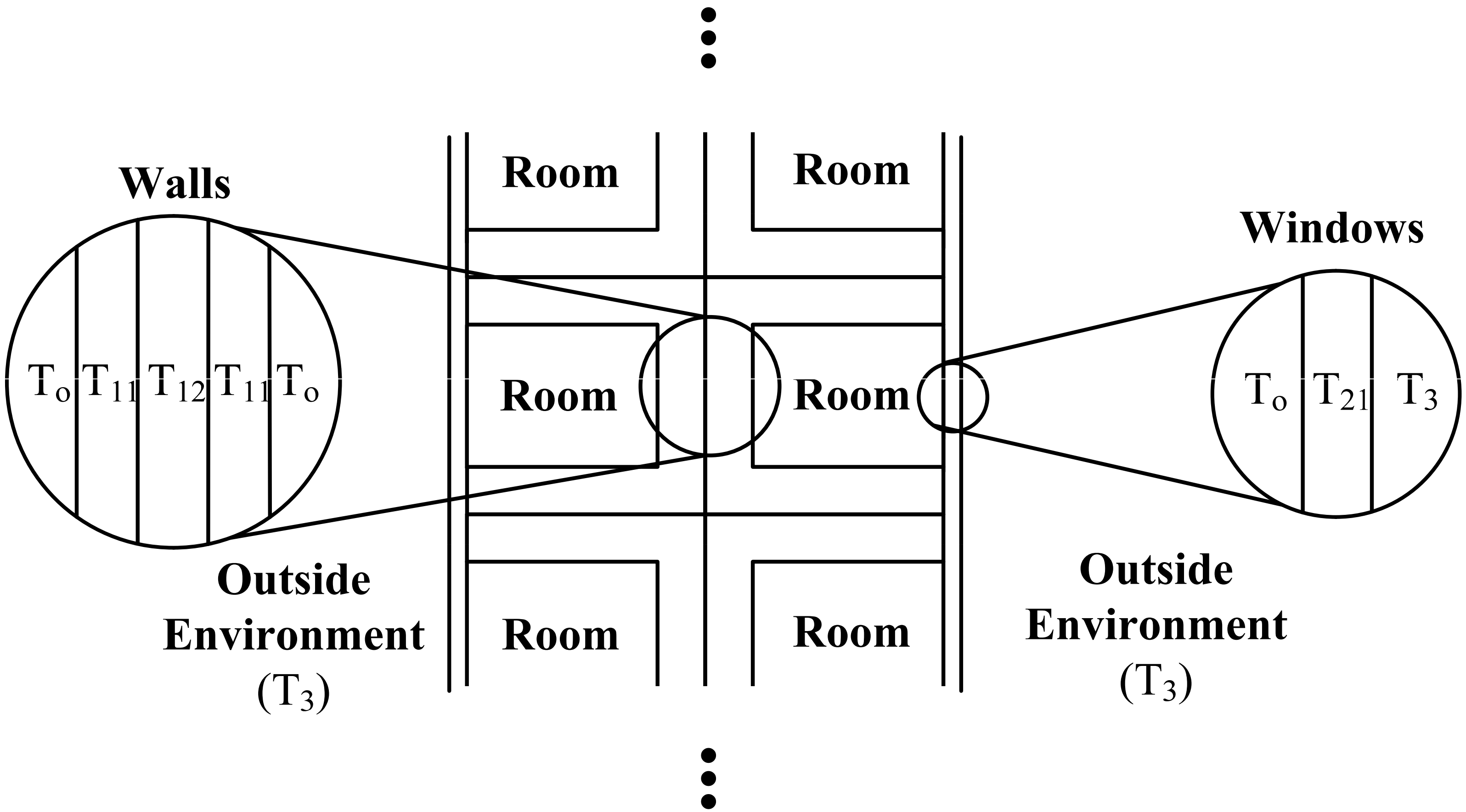
Economic Objective

$$g\left(x,u,w\right) = P_C C_e$$





5 State Example Building



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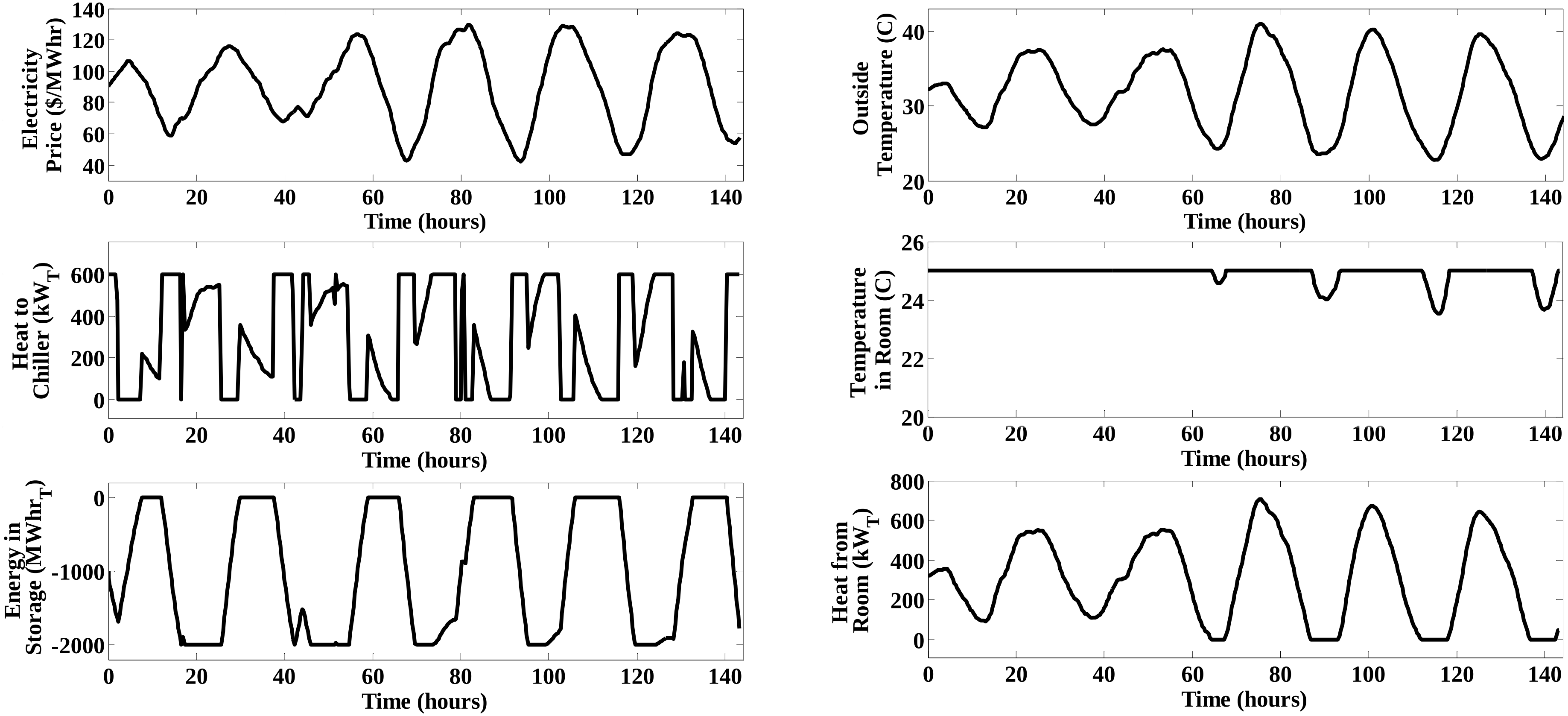
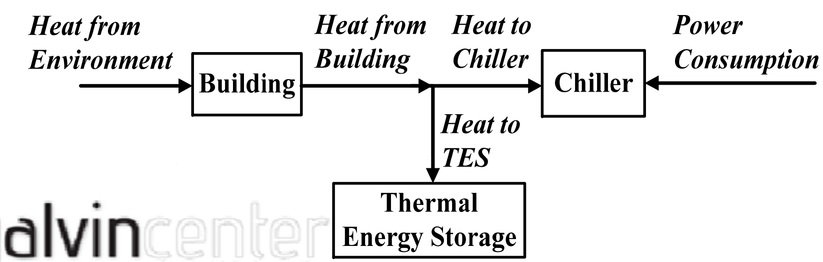
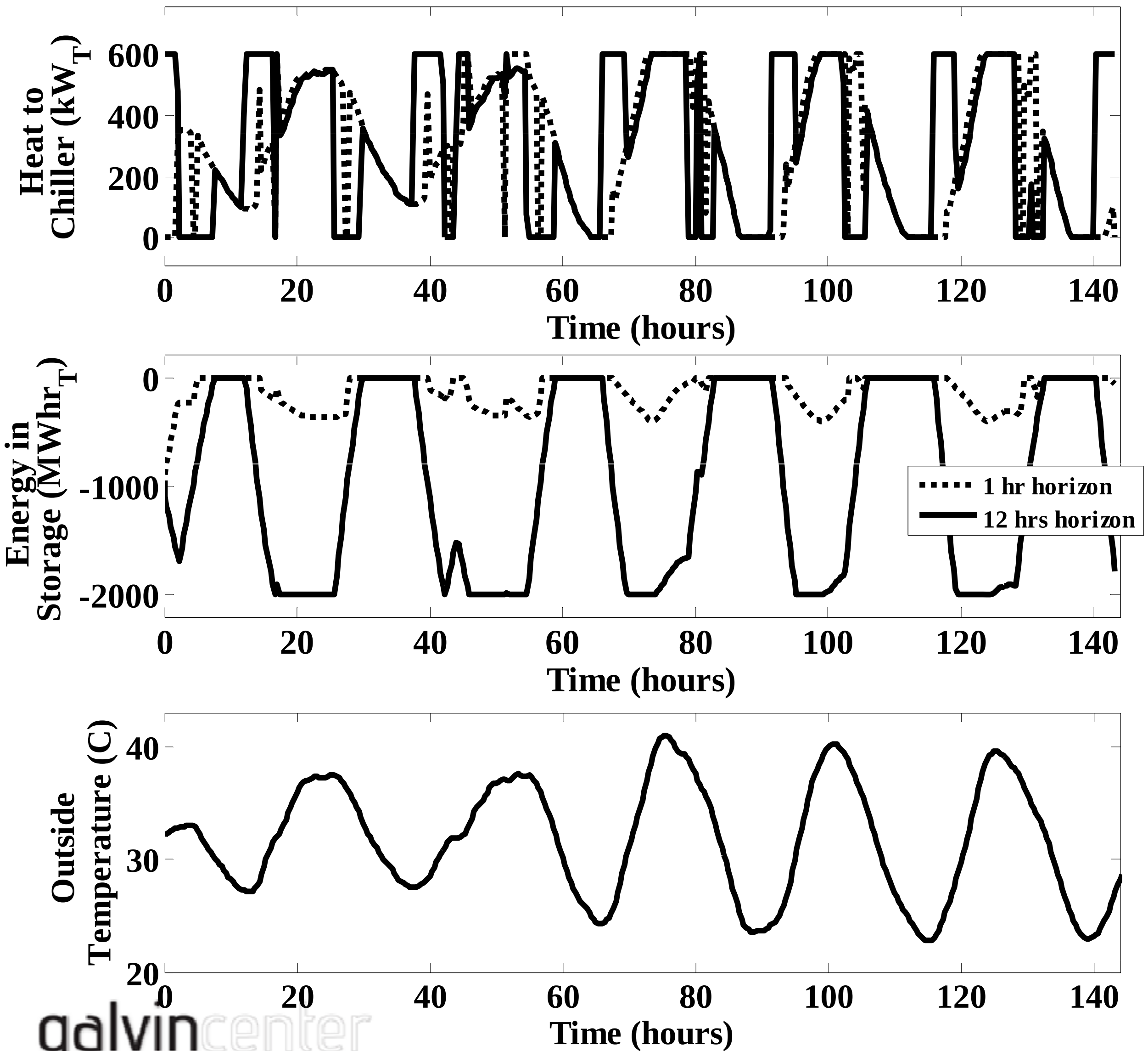
EMPC Simulation





Note: EMPC Prediction Horizon is 12 hours

Impact of Horizon Size on EMPC



Simulation Times:

T = 12 hr: 21504 sec

T = 1 hr: 2.6 sec

Operating Costs:

T = 12 hr: $746

T = 1 hr: $845

Economic Linear Optimal Control

Recall definition of Average Expenditures

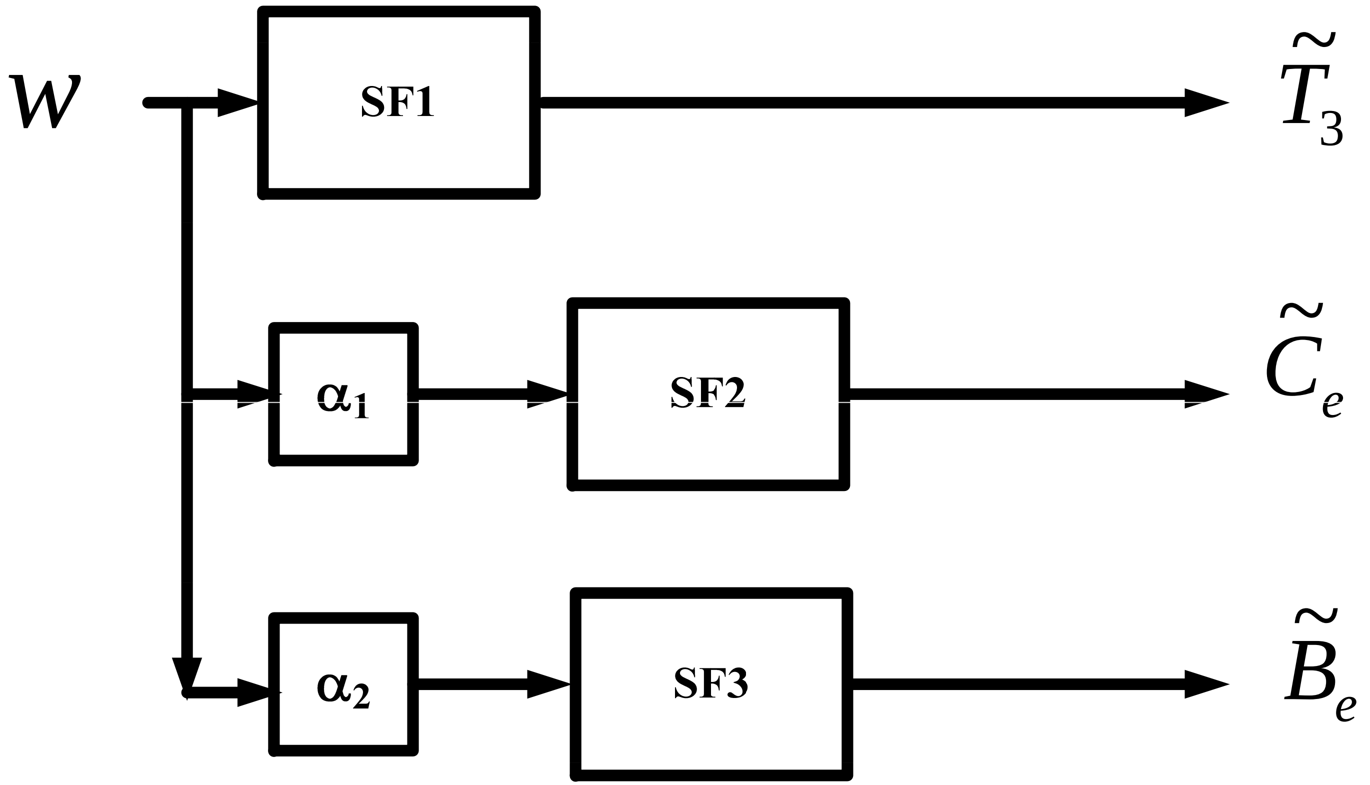
$$\overline{R} = \lim_{T\to\infty}\left\{\frac{1}{T}\int_0^T C_e P_c dt\right\} = E[C_e P_c]$$

ELOC enforces: $\tilde{P}_c = \alpha_1 \tilde{C}_e + \alpha_2 \tilde{B}_e$ and $E[\tilde{C}_e \tilde{B}_e] = 0$

($\tilde{P}_c = P_c - \overline{P}_c$ and $\tilde{C}_e = C_e - \overline{C}_e$)

Then:
$$\begin{aligned}\overline{R} &= E[\tilde{C}_e \tilde{P}_c] + \overline{C}_e \overline{P}_c \\ &= E[\alpha_1 \tilde{C}_e^{\,2}] + \overline{C}_e \overline{P}_c \\ &= \alpha_1 \Sigma_{Ce} + \overline{C}_e \overline{P}_c\end{aligned}$$

Disturbance Modeling



$$\dot{x} = Ax + Bu + Gw \qquad G = (G_0 + \alpha_1 G_1 + \alpha_2 G_2)$$

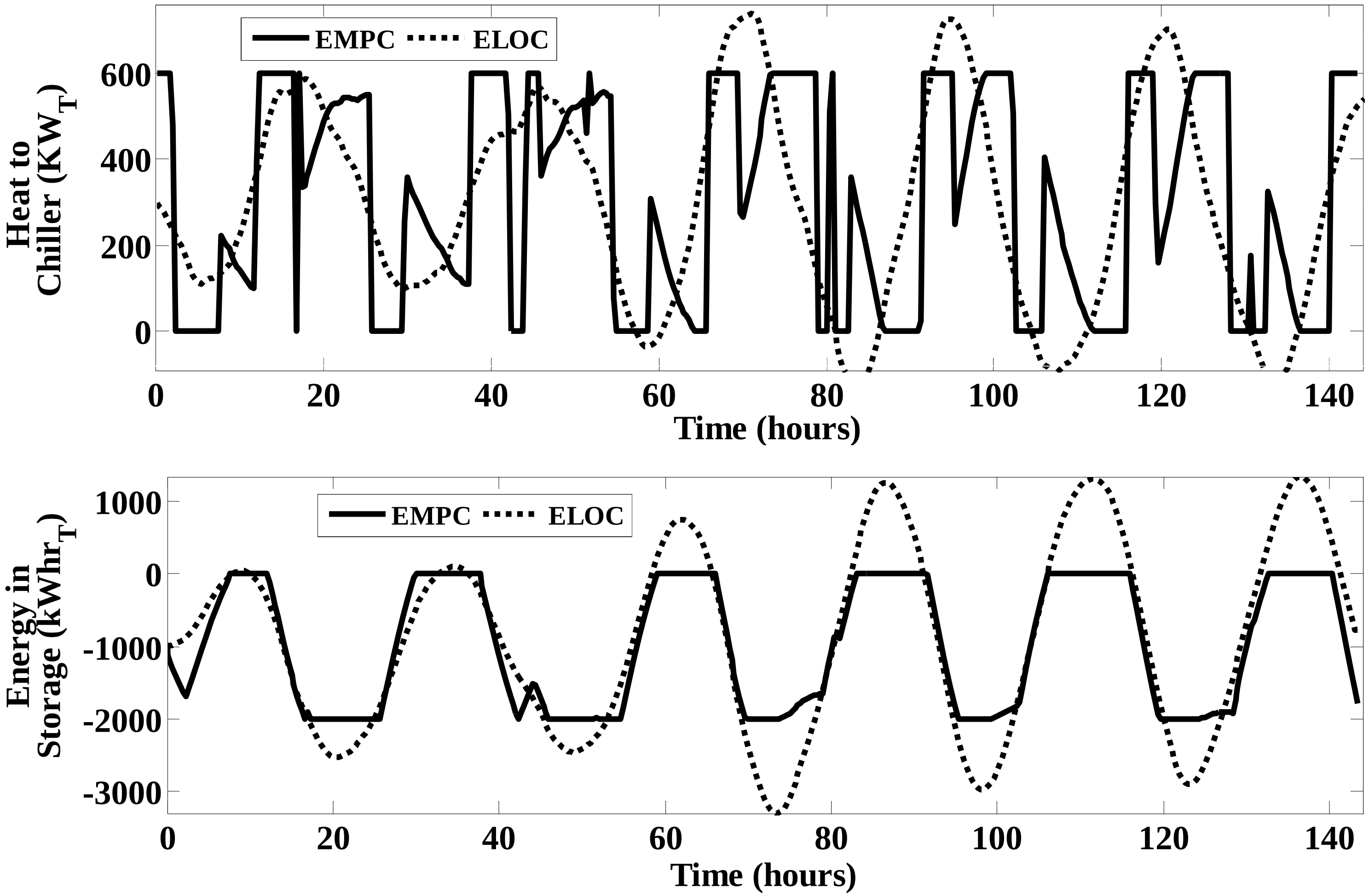
ELOC Synthesis

$$\min_{\substack{L,\Sigma_x \geq 0,\zeta_j,\sigma_j,\\ \alpha_1,\alpha_2}} \left\{\alpha_1 \Sigma_{Ce} + \overline{C}_e \overline{P}_c\right\}$$

$$\left.\begin{aligned}
& s.t. \\
& 0 = (A+BL)\Sigma_x + \Sigma_x (A+BL)^T + G S_w G^T \\
& G = G_0 + \alpha_1 G_1 + \alpha_2 G_2 \\
& \zeta_j = \rho_j (D_x + D_u L)\Sigma_x (D_x + D_u L)^T \rho_j^{\ T} \\
& \sigma_j = \sqrt{\zeta_j} \\
& 2\sigma_j < z_j^{\max} \quad \text{and} \quad 2\sigma_j < -z_j^{\min},\ j = 1 \ldots n_z
\end{aligned}\right\} \Rightarrow u = Lx$$

This problem can then be converted to a Convex Optimization Problem

Comparison of EMPC and ELOC



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Inverse Optimality and Infinite Horizon Unconstrained EMPC

$$u = Lx \Rightarrow \quad \min_{x,u} \left\{ \int_{t}^{t+T} \left(x'Qx + u'Ru \right) d\tau + x'(T)Px(T) \right\}$$

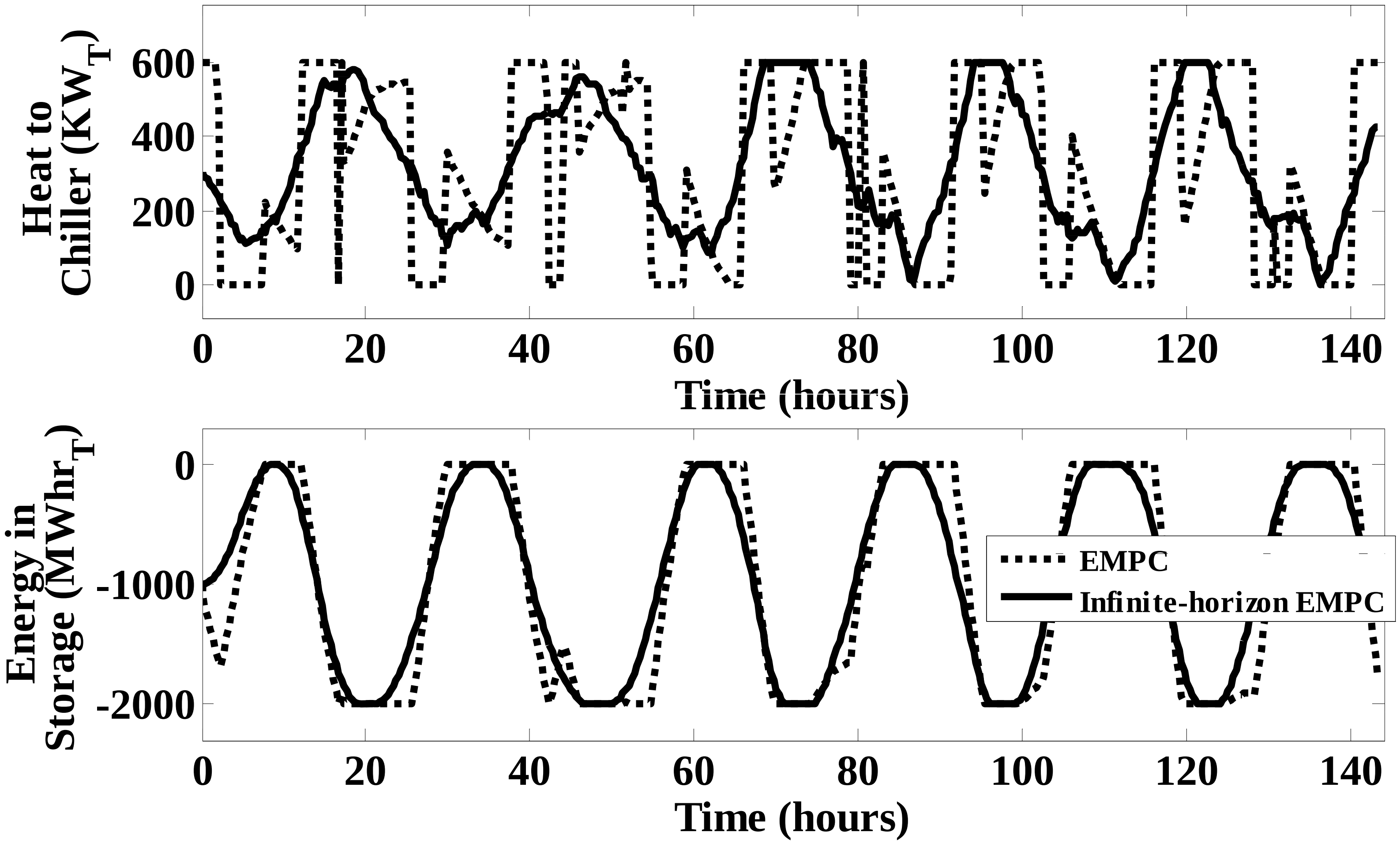
$$s.t. \quad \dot{x} = Ax + Bu$$

Infinite Horizon EMPC (IH-EMPC)

$$\min_{x,u}\left\{\int_{t}^{t+T}\left(x'Qx+u'Ru\right)d\tau+x'(T)Px(T)\right\}$$

$$\begin{aligned} s.t.\quad & \dot{x}=Ax+Bu \\ & z=D_x x+D_u u \\ & z^{\min}\le z\le z^{\max} \end{aligned}$$

Comparison of EMPC and IH-EMPC



Prediction horizons are: 12 hours EMPC and 1 hour for IH-EMPC

Comparison of EMPC and IH-EMPC

Simulation Times:

- EMPC $T = 12$ hr:
  21504 sec
- IH-EMPC $T = 1$ hr:
  2.6 sec

<u>99.987%</u> reduction in computational effort

Operating Costs:

- EMPC $T = 12$ hr:
  $746
- IH-EMPC $T = 1$ hr:
  $774

<u>3.75%</u> increase in operating costs

Acknowledgements

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  - David Mendoza-Serrano (paper co-author)
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Conclusions

- EMPC provides desired economic performance
- However, EMPC has challenges:
  - Bang-bang actuation and chattering
  - Large computational effort
  - Inventory creep for small horizons
- Infinite Horizon EMPC shown to:
  - Reduce bang-bang and chattering
  - Virtually no inventory creep for small horizons
  - Small computational effort for small horizons