Energy Pricing and Dispatch under Demand Response and Market Price Uncertainty

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1. Structure of the retail market

- The retailer purchases energy at a wholesale market as a price-taker, and sells the energy to consumers as a price-maker.
- Retailer owns a storage unit.
- Energy price at the wholesale market is uncertain.
- Consumers adjust their demands according to the retail price.
2. Mathematical formulation

A two-stage two-level decision framework

Stage 1: Day-ahead Setting retail price

\[
\max_{f, y} \sum_i f^T y^i (f) \\
\text{s.t. } f \in P
\]

Stage 2: Real-time Energy dispatch

\[
+ \min_{c \in W} \max_x c^T x \\
\text{s.t. } Ax + By \leq b
\]

Upper level 
Retailer

Lower level 
Consumers

**Upper level Retailer**

\[ p \]

\[ y^i \]

\[ y^i = \arg \min \hat{y}^i f^T \hat{y}^i \\
\text{s.t. } \hat{y}^i \in C_i^P \]

**Lower level Consumers**

\[ y^i \]
2. Mathematical formulation

\[
\begin{align*}
\max_{Pr_{\text{offer}} \in P, \{ p_{it} \} \in O(Pr_{\text{offer}})} & \quad \sum_{t=1}^{N_T} \sum_{i=1}^{N_C} Pr_{t}^{offer} p_{it} + \\
\min_{Pr \in W \{ E^b, E^s, r^c, r^d, S \} \in X} & \quad \max_{t} \sum_{t=1}^{N_T} \left( Pr_{t} - \varepsilon \right) E_t^s - \left( Pr_{t} + \varepsilon \right) E_t^b
\end{align*}
\]

\[
P = \left\{ Pr_{\text{offer}} \mid Pr_{t}^{\text{min}} \leq Pr_{t}^{\text{offer}} \leq Pr_{t}^{\text{max}}, \forall t \right\}
\]

\[
W = \left\{ Pr \mid \sum_{t} \left( Pr_{t} - Pr_{t}^{f} \right) / Pr_{t}^{v} = 0 \right\}
\]

\[
\sum_{t} \left| Pr_{t} - Pr_{t}^{f} \right| / Pr_{t}^{v} \leq \Gamma
\]
2. Mathematical formulation

\[ O(Pr^{offer}) = \{ p = \{ p^i = \{ p_{it} \}, \forall t \}, \forall i \mid \]

\[ p^i = \arg \min_{\hat{p}_{it} \in C_i^p} \sum_{t=1}^{N_T} Pr^{offer} \hat{p}_{it} \]

\[ C_i^P = \{ \hat{p}^i \mid \hat{p}_{it} \leq P_m^i, \forall t \in T_a : \rho_{it} \]
\[ \hat{p}_{it} \geq 0, \forall t \in T_a, \hat{p}_{it} = 0, \forall t \notin T_a : \theta_{it} \]

\[ \sum_t \hat{p}_{it} = Q_i^d, \mu_i \}

\[ X = \{ (E^b, E^s, r^c, r^d, S) \mid \]

\[ 0 \leq r_t^c \leq ST_c^m, 0 \leq r_t^d \leq ST_d^m, \forall t, 0 \leq S_t \leq ST_c^e, \forall t \in \{2, ..., N_T - 1\} \]

\[ S_t = S_{t-1} + \eta^c r_t^c - r_t^d / \eta^d, \forall t \in \{2, ..., N_T\}, S_1 = ST_0^e, t \in \{1, N_T\} \]

\[ \sum_{i=1}^{N_C} p_{it} + r_t^c - r_t^d = E_t^b - E_t^s, \forall t, E_t^b \geq 0, E_t^s \geq 0, \forall t \} \]
3. Solution method

A polyhedral representation of \( W \)

\[
W = \left\{ \Pr \left| \begin{array}{c}
Pr_t^l \leq Pr_t \leq Pr_t^u, \forall t \\
\sum_{t} (Pr_t - Pr_t^f)/Pr_t^v = 0 \\
\sum_{t} \left| Pr_t - Pr_t^f \right|/Pr_t^v \leq \Gamma \\
Pr \end{array} \right. \right\}
\]

\[
= \left\{ \Pr \left| \begin{array}{c}
Pr_t^l \leq Pr_t \leq Pr_t^u, \forall t : \eta_t^l, \eta_t^u \\
\sum_{t} (Pr_t - Pr_t^f)/Pr_t^v = 0 : \xi \\
\exists u, \sum_{i=1}^{N_T} u_t \leq \Gamma : \zeta \\
-\xi \leq \left( Pr_t - Pr_t^f \right)/Pr_t^v \leq \xi, \forall t : \lambda_t^l, \lambda_t^u \\
\right. \right\}
\]
3. Solution method

Second stage problem transformation

\[
\min \max_c c^T x, \quad c = [-\Pr - 1\varepsilon, \Pr + 1\varepsilon, 0]
\]

\[s.t. \ D\Pr + Fu \geq d, \ Ax \leq b - By\]

For a fixed bidding strategy, the worst-case market price solves an LP

\[
\min_c c^T x = \min (x_2 - x_1)\Pr_{Pr,u}
\]

\[s.t. \ D\Pr + Fu \geq d\]

Dual

\[\max d^T \lambda \quad \text{Dual}\]

\[s.t. \ F^T \lambda = 0, \ \lambda \geq 0\]

\[D^T \lambda = x_2 - x_1\]
3. Solution method

Second stage problem transformation

The linear min-max problem is equivalent to an LP

\[
\begin{align*}
\min_{c} \max_{x} c^T x \\
\quad \text{s.t.} \quad DPr + Fu & \geq d \\
\quad Ax & \leq b - By \\
\quad c & = [-Pr \pm \varepsilon, Pr \mp \varepsilon, 0]
\end{align*}
\]

\[
\begin{align*}
\max_{x, \lambda} d^T \lambda \\
\quad \text{s.t.} \quad F^T \lambda & = 0, \lambda \geq 0 \\
\quad D^T \lambda & = x_2 - x_1 \\
\quad Ax & \leq b - By
\end{align*}
\]

- This conclusion holds for polyhedral uncertainty in objective function.
3. Solution method

Lower-level problem transformation

\[
\text{min } p^{iT} \text{Pr}^{\text{offer}} \\
\text{s.t. } H^i p^i \leq h^i
\]

primal-dual optimality conditions

\[
\text{Pr}^{\text{offer}} p^i = h^T \mu \\
H^i p^i \leq h^i \\
\mu^i \leq 0, \ H^{iT} \mu^i = 0
\]

The optimization problem can be replaced by explicit constraints
3. Solution method

Linearizing bilinear terms

Apply binary expansion on the retail price

\[ \Pr_{t}^{offer} = \Pr_{t}^{min} + \Delta_{t}^{s} \sum_{l=1}^{K} 2^{l-1} z_{l}^{t}, \ \forall t \]

Introduce new variable \( v_{it}^{l} = p_{it} z_{l}^{t}, \ \forall i, t \)

The bilinear terms can be linearized as

\[ \Pr_{t}^{offer} \cdot p_{it} = \Pr_{t}^{min} \cdot p_{it} + \Delta_{t}^{s} \sum_{l=1}^{K} 2^{l-1} v_{it}^{l}, \ \forall i, t \]

\[ 0 \leq p_{it} - v_{it}^{l} \leq P_{i}^{m} (1 - z_{l}^{t}), \ \forall i, t, l \]

\[ 0 \leq v_{it}^{l} \leq P_{i}^{m} z_{l}^{t}, \ \forall i, t, l, \ z_{l}^{t} \in \{0,1\}, \ \forall t, l \]

The number of binary variables are independent of the number of consumers
3. Solution method

The Equivalent MILP

\[
\begin{align*}
\max & \sum_{i=1}^{N_c} (p_i^T \Pr_{\min} + 1^T V_i \Delta) + d^T \lambda \\
\text{s.t.} & \quad \{\Pr_{\text{offer}} = \Pr_{\min} + Z \Delta\} \in P \\
& \quad F^T \lambda = 0, \lambda \geq 0, D^T \lambda = x_2 - x_1 \\
& \quad Ax \leq b - By, \quad y = \{p_i\}, \forall i \\
& \quad \Pr_{\min} p_i + 1^T V_i \Delta = h^T \mu^i \\
& \quad H^i p_i \leq h^i, \mu^i \leq 0, H^T \mu^i = 0 \\
& \quad 0 \leq p_i^1 - V_i \leq P_i^m (1 - Z), \forall i \\
& \quad 0 \leq V_i \leq P_i^m Z, \forall i, Z \in \{0,1\}^{T \times L}
\end{align*}
\]
3. Solution method

Pareto efficiency test

- The risk-aversive dispatch strategy only accounts for the worst-case market price realization.
- There may be another strategy that can improve the retailer’s profit for at least one scenario of market price without deteriorating it in all other scenarios.

\[
PE = \max_{\gamma^1, \gamma^2, \gamma^3, \gamma^4, \gamma^5} \left( \Pr^f - 1\epsilon \right)^T \gamma^1 - \left( \Pr^f + 1\epsilon \right)^T \gamma^2 \\
\text{s.t. } (\gamma^1, \gamma^2) \in W^* \\
\{E^b_* + \gamma^1, E^s_* + \gamma^2, r^c_* + \gamma^3, r^d_* + \gamma^4, S_* + \gamma^5 \} \in X
\]
3. Solution method

Pareto efficiency test

$W^*$ is the dual cone of the polytope $W$

\[ W^* = \{(\gamma^1, \gamma^2) \mid (Pr - \mathbf{1} \varepsilon)^T \gamma^1 - (Pr + \mathbf{1} \varepsilon)^T \gamma^2 \geq 0, \ \forall Pr \in W \} \]

\[ = \{(\gamma^1, \gamma^2) \mid \exists \{ \eta^l_t, \eta^u_t, \lambda^l_t, \lambda^u_t, \xi, \zeta \}, \text{such that} \]

\[ \eta^l_t \geq 0, \ \eta^u_t \leq 0, \ \lambda^l_t \geq 0, \ \lambda^u_t \leq 0, \ \forall t, \ \zeta \leq 0 \]

\[ \gamma^1_t - \gamma^2_t - \eta^l_t - \eta^u_t - \lambda^l_t/Pr^v_t - \lambda^u_t/Pr^v_t - \xi/Pr^v_t = 0, \forall t \]

\[ -\lambda^l_t + \lambda^u_t - \zeta = 0, \ \forall t \]

\[ \sum_{t=1}^{N_T} (Pr^f_t \eta^l_t + Pr^u_t \eta^u_t + (\lambda^l_t + \lambda^u_t) Pr^f_t/Pr^v_t) \]

\[ + \zeta \sum_t Pr^f_t/Pr^v_t + \zeta \Gamma \geq \varepsilon \sum_{t=1}^{N_T} (\gamma^1_t + \gamma^2_t) \} \]
3. Solution method

Pareto efficiency test

- Because 0 is a feasible solution, PE≥0.
- If PE = 0, the current dispatch strategy $x^* = \{E^b_*, E^s_*, r^c_*, r^d_*, S_*\}$ is already non-dominated.
- If PE > 0, $z^* = x^* + \gamma^*$ dominates $x^*$ and $z^*$ is non-dominated.

4. Case studies

Retail and market price

<table>
<thead>
<tr>
<th>Period</th>
<th>Price (USD/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecast</td>
</tr>
<tr>
<td></td>
<td>Lower bound</td>
</tr>
<tr>
<td></td>
<td>Upper bound</td>
</tr>
<tr>
<td>Worst case</td>
<td>Offering price</td>
</tr>
</tbody>
</table>

Graph showing the comparison of different price scenarios over periods.
4. Case studies

Energy contract
4. Case studies

Operation of storage unit
4. Case studies

Profit enhancement

- Profit under robust strategy
- Profit under Pareto robust strategy
Thanks!